On the origin of the double-bump IS spectrum

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Abstract

It is stated in an often-cited review article that the double-bump character of an IS ion-line spectrum is due to the Doppler-shift of the radar radiation as it scatters from upward-going and downward-going ion-acoustic waves.[1] The size of the shift corresponds to the phase velocity of the wave. The problem with this view is that, unlike the normal Doppler-shift associated with the bulk motion of the plasma, the primary scattering objects, the plasma electrons, are not moving systematically with the wave's velocity; rather, they can be considered to be at rest. This note tries to clarify, from a very elementary, qualitative, point of view, what actually is happening. The shift's mechanism is simpler than in the standard Doppler-shift. The shift is basically due to a target-selection effect. The dominant contributions to the reception come from the locations of density wave crests, and when the phase of the wave changes, different dominant ranges, with different phase delays, become selected. That is, the targets are at rest and have zero Doppler, but different sets of targets become selected when the wave propagates. This causes an extra change of reception phase at the rate of the angular frequency of the density wave. The note also dwells in detail on the "Bragg-condition", or why and when a density wave can enhance the received field in the first place.

Scattering from a single electron

We are ignoring all characteristics of the radiation except phase, and are considering only the backscatter situation. We are also considering a smallish volume at a long range r, so that we will be able to ignore all distance-dependent factors also. We write the (phase-part of the) electric field at transmission as

$$E_0(t) \ \sim \mathrm{e}^{i\omega_{\mathrm{rad}} t}\,, \tag{1}$$

where ω_{rad} is the radar frequency. We consider scattering from a single electron, moving at the plasma's bulk velocity v_{p} along the radar beam so that its range is

$$R(t) = r_0 + v_{\mathrm{p}} t\,. \tag{2}$$

The electric field $E(t)$ at reception is proportional to the transmitted field at a delayed time t',

$$E(t) \sim E_0(t')\,. \tag{3}$$

From the space-time geometry of the situation, Fig. 1, the delayed time is

$$t' = \frac{c - v_{\mathrm{p}}}{c + v_{\mathrm{p}}} t + \frac{2r_0}{c + v_{\mathrm{p}}} = (1 - 2\frac{v_{\mathrm{p}}}{c})t - \frac{2r_0}{c}(1 - \frac{v_{\mathrm{p}}}{c}) + \mathcal{O}(\frac{v_{\mathrm{p}}}{c})^2\,. \tag{4}$$

[1] Beynon and Williams, Rep. Prog. Phys, Vol 41, Number 6, 1978, p.917: "If the signal is scattered by an upward traveling wave it will experience a Doppler-frequency shift: $\Delta f = -2V/\lambda$ [= . . .]"

The pulse propagation delay PPD is

$$\mathrm{PPD} \equiv t - t' = \frac{2R(t)}{v_\mathrm{p} + c}\,. \tag{5}$$

The Doppler-shift

We are going to combine at reception electric fields arriving from a set of electrons within the small volume (in which there is some hope of density waves staying coherent). Moreover, apart from the uniform bulk plasma motion with the velocity v_p, we assume the electrons do not not have any systematic motion. Then we can always choose the time origin so that for the purpose of the phase bookkeeping, the second term in Eq. (4) can be replaced by $2r/c$, with r being the range at the time of scattering. The received field from a single electron at distance r becomes, to first order in v_p/c,

$$E(t) \sim E_0(t') \sim \mathrm{e}^{i(\omega_\mathrm{rad}+\omega_\mathrm{D})t} \cdot \mathrm{e}^{-iKr}\,, \tag{6}$$

where

$$\omega_\mathrm{D} = -\frac{2v_\mathrm{p}}{c}\omega_\mathrm{rad} \tag{7}$$

is the backscatter Doppler-shift, and K is twice the radar wavenumber,

$$K = \frac{2\omega_\mathrm{rad}}{c} = 2\,\frac{2\pi}{\lambda_\mathrm{rad}} = 2k_\mathrm{rad}\,. \tag{8}$$

The phase factor of the received field, Eq. (6), is reasonable. The first term takes care of the Doppler-motion, the second term, $k_\mathrm{rad} \times 2r$, does not depend on the uniform motion but just represents the total phase change of the (non-Doppler-shifted) radar wave along its flight path. When we below consider waves of plasma electron density, we can compute the frequency changes as if the plasma where at rest, ignoring the common Doppler-term from the equations. That is, when we refer to the density wave's phase velocity, v_w, we mean the phase velocity in the rest frame of the plasma; the bulk velocity is taken care by the Doppler-term in Eq. (6).

The value given in Eq. (7) for the backscatter Doppler-shift follows from the scattering geometry and the requirement Eq. (3). A re-derivation of the Doppler-shift, directly from the geometry of the space-time diagram, is given in the caption of Fig. 2. Ultimately, the physics of the backscatter Doppler-shift must be justified from Maxwell equations, but an intuitive explanation is given, for example, in Tuomo Nygren's book[2]. It is stated that the Doppler-shift results from two factors:

(P1) Due to its motion in the radar frame, in its own rest frame the electron hits radiation wavefronts at different rate than if it were at rest. In its rest frame, it emits radiation with this altered frequency. This gives one factor of $\frac{v_\mathrm{p}}{c}\omega_\mathrm{rad}$ to Eq. (7).

(P2) Due to the motion of the radiation source, the distance between the emitted wavefronts changes. This gives another factor of $\frac{v_\mathrm{p}}{c}\omega_\mathrm{rad}$.

[2] An unpublished early version of the coming, extended book; not the published book from 1996.

We will show below how the strong semi-coherent scatter that is responsible for the ion-line (and similarly for the plasma lines) is due to a pattern of spatially varying electron density. It is not unreasonable to imagine that if the pattern is moving, we should be able to draw for it a space-time diagram similar to Fig. 2, where the world-line is the world-line of the pattern and the velocity is the pattern's velocity v_{w}, which is the phase velocity of the wave. The frequency-shift follows essentially from the diagram, so heuristically we expect a frequency shift ω_{w} of the form of Eq. (7), with v_{p} replaced by v_{w}.

One can call the wave-related frequency shift ω_{w} a Doppler-shift if one must, but we clearly cannot apply the argument P1–2 here. The primary scatterers are still the electrons, but they are *not* moving with the velocity v_{w}, not in any sense of the word. Worse, we get frequency shifts $\pm\omega_{\mathrm{w}}$ from a thing that looks even less uniformly-moving than a traveling wave, namely, from a standing way, such as a plasma oscillation. Of course, a standing wave is just two opposite-moving traveling waves, so this situation is only marginally worse. But something else should take the place of P1–2 to illuminate the physics.

Scattering from a density wave

When there are multiple scattering electrons in the small volume V of interest, the electric fields add up to give the (volume's contribution to the) received field

$$E(t) \sim (\int_V N(\mathbf{r},t)\mathrm{e}^{-iKr}d\mathbf{r})\,,\times\,\mathrm{e}^{i(\omega+\omega_D)t} \tag{9}$$

where $N(\mathbf{r},t)$ is the plasma rest-frame electron number density. We will call Eq. (9) the superposition integral in this note. We will only consider wavelike structures of N in the beam direction, and will subsequently write the superposition integral only as 1-dimensional integral in the beam direction. This can be done because any oblique plane wave (-vector) can be composed into beam-parallel and beam-perpendicular waves. A plane wave perpendicular to the beam has constant density in the beam direction, and as we show below, a constant density in beam direction will not contribute to the received E. So we need only consider beam-parallel structures. These have constant density in the perp direction, and so the perp direction can be taken out of the integral.

The scattering volume we are considering must be small, of the size of coherence length of the longitudinal structures. Only then it makes sense to model the density by a single sinusoid.

The received electric field due to such a coherent structure is a deterministic function. The actually received electric field at any given time, from the whole illuminated volume, could be modeled as an incoherent sum of a very large number of such coherent contributions, and thus the actually received field must ultimately be modeled as a random process. The individual contributions to the total field can be assumed to have roughly similar power spectra. These in turn are essentially the squared moduli of the spectra (Fourier transforms) of the deterministic $E(t)$, and thus have qualitatively similar shape to it. For the purpose of this qualitative note, Eq. (9) is good enough.

We ask under which conditions the superposition integral will become large. The crucial observation is that the exponential factor oscillates rapidly all along the unit circle. For any sizable volume V, if N is spatially constant so that it can be taken out of the integral, the phase factors mostly cancel, and the integral becomes small.

Thus, there is little net backscatter from the volume if the density is nearly constant.[3] The physical reason is that at any moment of time in reception, there is full destructive interference along the world-line of the received plane wave, as illustrated by the space-time diagram of Fig. 3. Thus, it is only density fluctuations δN that can produce some net scattering,

$$N(\mathbf{r}, t) = N_0 + \delta N(\mathbf{r}, t)\,. \tag{10}$$

Because the plasma ultimately is discrete and its elementary constituents are in random thermal motion, the situation when plasma can be said to have a ”nearly constant density”, and to what degree, is both probing wavelength and plasma temperature dependent. A density that appears constant with one wavelength and one temperature, will look fluctuating with sufficiently shorter wavelength at that temperature, or at sufficiently higher temperature when using the same wavelength. This behaviour is quantified via the Debye length λ_{Debye}, which depends both on the density and on the temperature, $\lambda_{\mathrm{Debye}} \propto \sqrt{T/N}$. Inspected with resolution significantly better than the Debye-length, the elementary scatterers behave truly independently (move randomly). In this case, the density is always fluctuating randomly, and there will always be some net scatter. This is the “truly incoherent” scatter, but that scatter is comparatively weak. It is the truly incoherent scatter that gives rise to the electron line component of the full IS spectrum when the plasma is studied with short wavelengths.

But with radar wavelengths longer than the Debye-length, it is possible to enjoy enhanced scatter, due to suitable non-random density fluctuations. Even without the superposition integral Eq. (9) at hand, inspection of Fig. 3 already suggests the required arrangement. The sign-changing transmission divides spacetime into “plus” and “minus” stripes. With constant plasma density, these stripes contribute identically to the received signal at any given time, and thus cancel each other out. What we therefore need to do, is to modulate the density in a way matched to the stripes (that is, to the radar wavelength), so that full destructive interferece no more can occur. One way to ensure the required density pattern is to impose a time-independent sinusoidal density variation of wavelength $\lambda_{\mathrm{rad}}/2$ in the range direction, as shown in Fig. 4.

It is clear from the Fig. 4 and Fig. 5 that the wavelength λ_{w} of the fluctuation really must be matched to the radar wavelength as drawn, via $\lambda_{\mathrm{w}} = \lambda_{\mathrm{rad}}/2$. If this is not the case, the two involved sinusoids either go out of phase, and a messy “beat pattern” of phase results in any scattering region of space-time, and these phases tend to cancel each others out. This would occur if the sinusoids are only slightly non-matching as in Fig. 5 panels (c) and (g). If they are substantially non-matching, then one of these occur: If the fluctuation varies very rapidly compared to the plus-minus stripes of the radiation, the fluctuation chops each stripe to many individual plus-minus pieces, and these cancel each other out. If, on the other hand, the fluctuation is very slowly varying compared to the radar wavelength, we are back in the constant-density situation, and there again is not much net scatter.

In real plasma, there are no such frozen fluctuations, but time-dependent fluctuations, waves, occur naturally. In particular, the ion-line is associated with the electron density fluctuation occurring due to ion-acoustic waves. These are relatively low-frequency longitudinal electrostatic waves where the displacement from equilibrium is

[3] The only net scatter there is in the situation of constant density is forward scatter; in Eq. (9) forward scatter would show up as $K = 0$, and so the rapid oscillation does not occur. Physically: At reception in the forward direction all scatterers always add in phase, because all phase paths are of equal lengths. In all other directions, there will be a full collection of phase path lengths available.

due the thermally generated electron density variations and the overshoot due mainly to ion mass, so that the phase velocity $v_\mathrm{w} \approx \sqrt{k_\mathrm{B} T_i / m_i}$. There are wavelength-dependent corrections as well, so the waves are actually dispersive. In typical F-layer conditions, $v_\mathrm{w} \approx 1000 \ \mathrm{m\,s^{-1}}$, so that at EISCAT UHF with wavenumber $k_\mathrm{rad} \approx 20$ rad m^{-1}, the matching ion-acoustic wave's angular frequency is about $40{\times}10^3$ rad s^{-1}.

Physics of the the frequency shift when scattering from a density wave

We now come to the crux of this note. Given a wavelength-matched electron density fluctuation in the form of a traveling wave, what happens in reception? We expect a frequency shift ω_w equal to the Doppler-shift of an object moving with the wave's phase velocity. We will derive this result below from the superposition integral. But we can understand the result qualitatively by referring to the spacetime diagrams in Figures 4 and 6. Figure 4 suggests a strong connection between the phase of the fluctuation and the phase of the reception, which we state as proposition P3:

(P3) If, in the spirit of repeated trials, one changes the phase of a frozen fluctuation by a constant amount throughout the scattering volume, the change shows up in an equal change in the phase of the received field.

The proposition holds because the reception depends linearly on the fluctuation. That is, because $E(t) = a \int_V g(t, \mathbf{r}) N(t, \mathbf{r}) d\mathbf{r}$, it follows that if $N \to \mathrm{e}^{i\phi} N$, then $E \to \mathrm{e}^{i\phi} E$. It does not matter *why* the phase changes. In principle, it could change because direct manipulation of the electron density, say by cleverly streaming charges in and out of the radar beam in a beam-perpendicular direction (an operation which we would normally not expect to cause any radial "Doppler-shift"). Or we could just create and annihilate electrons in place. Or, more like the case in point, the fluctuation could change because there is a longitudinal density wave going through the region, in which case the phase of $\delta N(\mathbf{r})$ at each $\mathbf{r}$ changes at the rate ω_w, the angular frequency of the wave.

The rate of change of the received field has up to three distinct contributing factors, each of different physical origin. First, as Fig. 6 illustrates, if the fluctuation is perfectly frozen, the reception phase anyway changes by the angular frequency ω_rad of the transmission. Second, if also the fluctuation is changing, it causes an extra change of phase in reception, and this change ω_w must be added on top of ω_rad. And third, if there is bulk plasma motion, one needs to throw in the Doppler-shift ω_D of the bulk motion also.

Derivation of the frequency shift due to a density wave

We inspect the superposition integral in Eq. (9) (now only in radial direction, z). We note that we get a large value only when the fluctuation δN is able to cancel the exponential factor. That is, it should have a sinusoidal spatial form with a wavelength that is matched to the radar wavelength by

$$\delta N(t, z) = n_0(t) \mathrm{e}^{+iKz}\,, \tag{11}$$

with K given in Eq. (8), $K = 2k_\mathrm{rad}$. All other forms would leave a more or less rapidly oscillating exponential.[4] The condition Eq. (11) for strong scatter—strong constructive

[4] The mathematical result behind this rather woolly claim is the Riemann-Lebesgue lemma, which states that for any fixed interval L and any reasonable f, $\lim_{K\to\infty} \int_L f(z) \mathrm{e}^{iKz} dz = 0$.

interference at reception— resembles the Bragg-backscatter condition of radiation scattering from crystals. For instance, given a radiation wavelength $\lambda_{\rm rad}$, we observe strong backscatter only if the lattice spacing D is

$$D = \frac{\lambda_{\rm rad}}{2}. \tag{12}$$

When inspecting a crystal, one typically would sweep over a range of frequencies to find what is the lattice spacing. In the radar case, the frequency is more or less fixed for a given radar, and we say that the radar can only "see" the target well if the target has coherent structures with the spatial period given by Eq. (12).

A natural way that coherent periodic structures of type Eq. (11) can occur is longitudinal density wave motion. These may be traveling waves or they can as well be standing waves. The latter are of sums of traveling waves, moving in opposite directions.

Consider an upward traveling density wave, with angular frequency $\omega_{\rm w}$, of the electron density fluctuation,

$$\delta N = \cos(\omega_{\rm w} t - k_{\rm w} z)\,, \tag{13}$$

with phase velocity $v_{\rm w}$, so that

$$\omega_{\rm w} = k_{\rm w}\, v_{\rm w}\,. \tag{14}$$

We use an explicitly real-valued plane wave in Eq. (14), instead of $\mathrm{e}^{i(\omega_{\rm w} t - k_{\rm w} z)}$, because the complex presentation would, a little too conveniently, skip over a complication that I want to point out. Even though it is right and proper to use a single complex wave (here the radar wave) in linear equations in place of the actual physical wave, the situation is not clear-cut if one needs to multiply the waves.

We know from Eq. (11) that for this wave to cause strong signal at reception, it must have a wave number about twice the wavenumber of the radiation, so we require

$$k_{\rm w} \approx K\,. \tag{15}$$

What is the frequency of the received electric field? We write δN of Eq. (13) in terms of two complex exponentials and insert into the superposition integral Eq. (9), giving

$$E(t) \sim \int_L [\mathrm{e}^{i(\omega_{\rm w} t - k_{\rm w} z)} + \mathrm{e}^{-i(\omega_{\rm w} t - k_{\rm w} z)}]\mathrm{e}^{-iKz} dz \;\times\; \mathrm{e}^{(\omega+\omega_{\rm D})t} \tag{16}$$

$$= \int_L \mathrm{e}^{-i(k_{\rm w}+K)z} dz \;\times\; \mathrm{e}^{i(\omega+\omega_{\rm D}+\omega_{\rm w})t} + \int_L \mathrm{e}^{+i(k_{\rm w}-K)z} dz \;\times\; \mathrm{e}^{i(\omega+\omega_{\rm D}-\omega_{\rm w})t} \tag{17}$$

$$\approx \int_L \mathrm{e}^{+i(k_{\rm w}-K)z} dz \;\times\; \mathrm{e}^{i(\omega+\omega_{\rm D}-\omega_{\rm w})t}\,. \tag{18}$$

To get Eq. (18), we used Eq. (15) to drop the first term of Eq. (17) in comparison to the second. We conclude from Eq. (18) that the up-going wave leads to an extra downward frequency shift, in addition to the Doppler-shift due to bulk plasma motion, by the amount equal to the wave's angular frequency $\omega_{\rm w}$, which from Eq. (14), (15) and (8) can be written as

$$\omega_{\rm w} = \frac{2\omega_{\rm rad}}{c} v_{\rm w}\,. \tag{19}$$

By comparison with Eq. (7), the received field due to the upward moving wave has a frequency shift as if the radiation would have reflected from a plasma body moving uniformly with the phase velocity of the wave.

To get Eq. (18) from Eq. (17) we dropped the term containing a "fast varying" complex exponential $\exp(-i2Kz)$ in the integral. This seems to make sense, but why do we have such a term in the first place—the term would give a frequency shift of size $\omega_{\rm w}$ but in the "wrong" direction? The integral is trivially evaluated (as in Eq. (20), below), and then one notices that if the interval of integration L becomes infinite, or if L is a multiple of the matching wave-length $\lambda/2$, the integral vanishes without approximation. Does it mean that the term somehow corresponds to "edge-effects"? Or does it mean that the situation here is even more like mixing of a (complex) signal with a real mixing frequency, where both the up-shifted and downshifted mixing results do occur, but the unwanted mixing result is filtered out. In amplitude modulation, one normally gets two "sidebands", shifted away from the carrier by the modulation frequency, so maybe that is what happens here also, but I don't really know what to make out of this in physical terms.

In Eq. (18) the Doppler-shift $\omega_{\rm D}$ and the shift associated with the wave motion $\omega_{\rm w}$ combine to a total shift $\Delta\omega = \omega_{\rm D} + \omega_{\rm w}$. The total shift corresponds to the velocity $v_{\rm p} + v_{\rm w}$, the total phase velocity of the wave in the radar frame of reference.

Broadening of the spectral line

We have explicitly written the superposition integral to be over only a finite interval, L, because otherwise we cannot assume a coherent fluctuation. Of course, this is equivalent of writing the integral over infinite range, but using a boxcar cutoff function for the fluctuation. Physically, the cutoff correspond the wave being strongly damped. Either way, due to the finite interval of integration, the wavelength matching requirement needs be fulfilled only approximately. That is, there is a whole collection of fluctuation wavenumbers k_1 around $K = 2k_{\rm rad}$ that give reasonably strong net constructive interference at reception. The shorter wave coherence length L_K we have, the larger set of wavenumbers near K will be able to contribute to the signal.

Conversely, we can use the known shape of the ion-line to try to get on order-of-magnitude estimate on how large the typical damping distance might be. Very roughly, we imagine a typical double-bump spectrum of total width B to have the bump-centers, which presumably correspond to the exact-matching frequencies $\pm\omega_{\rm w}$, at positions $\pm B/4$. We evaluate the superposition integral Eq. (18) for a fluctuation of wavenumber k_1, of length L_1, to get

$$|E(t;k_1)| \sim \frac{|\sin[(k_1 - K)L_1/2]|}{|k_1 - K|}\,. \tag{20}$$

This has the first zero at k_1 such that $|k_1 - K| = \frac{2\pi}{L_1}$, or

$$|\frac{k_1}{K} - 1| = \frac{2\pi}{L_1}\,\frac{1}{K} = \frac{\lambda_{\rm rad}/2}{L_1}\,. \tag{21}$$

From the shape of the spectrum, we conclude that the contributing wavenumbers k_1 extend, very roughly, from zero to about as large as $2K$. This requires the coherence length to be quite short, $L_1 \lesssim \lambda_{\rm rad}/2$, or

$$L_1 \lesssim 1 \times \lambda_{\rm match}(K)\,. \tag{22}$$

That is, the density fluctuations must damp quite fast, within the size of the matching wavelength. This appears to be consistent with what we get by computing the decay half-time of the wave amplitude from a suitable Landau-damping formula, like the formula 5.96 of Bellan[5]. In typical ionospheric-F conditions at EISCAT UHF we find a decay half-time of about one wave period for all values of $T_\mathrm{i}/T_\mathrm{e}$ between about 1 and 6, as Fig. 7 shows.

Scattering as a filtering operation

When considering scattering from a density fluctuation it is possible to compute the received signal via a filtering operation. To realize that the superposition integral Eq. (18) in fact is a convolution integral, requires only a change of variable. The filtering picture brings no new physics, of course, but provides a handy way for computing the superposition integral, as we have done in Fig. 6.

One only needs to change from the radial variable z to an associated time variable t' via $z = \frac{c}{2}t' = \frac{\omega}{K}t'$, to get

$$E(t) \sim \int_L \mathrm{e}^{ik_1 z}\mathrm{e}^{i(\omega t - Kz)}dz \;\times\; \mathrm{e}^{-i\omega_1 t} \tag{23}$$

$$= \frac{c}{2}\int_{\frac{2L}{c}} \mathrm{e}^{i\frac{c}{2}k_1 t'}\mathrm{e}^{i\omega(t-t')}dt' \;\times\; \mathrm{e}^{-i\omega_1 t}\,. \tag{24}$$

The integral in Eq. (24) is a convolution integral of type $\int h(t')x(t-t')dt'$, where the filter h is

$$h(t) = \mathrm{e}^{i\frac{c}{2}k_1 t}\,, \tag{25}$$

which, taking into account the finite interval of integration, is a narrow-band band-pass, or notch-, filter, with the notch at position of the fluctuation frequency $(c/2)k_1$.

We now (for good or ill) re-cast the message of the previous section into the filtering picture. How narrow the filter passband is, depends on how large the coherent structure is. The larger L, the narrower is the notch, becoming a delta-function when $L \to \inf$. But when L is not large, the passband has some finite width, proportional to $1/L$, as implied by Eq. (20).

The finite passband implies that there is a whole collection of the density-fluctuation filters, of wavenumbers k_1 around the wavenumber $2k_\mathrm{rad}$, with a passband that contains the radar frequency, and so can contribute to the received field. Each filter brings with it its associated "mixing frequency" ω_1 which translates the filter output from ω_rad to $\omega_\mathrm{rad} + \omega_1$. We conclude that the spectrum $\mathcal{E}(\omega)$ of $E(t)$ originating from the small volume of coherent scattering, displays a continuous collection of frequencies around $\omega_\mathrm{rad} \pm \omega_1$.

More formally the "collection" means a Fourier decomposition of the fluctuation, so that the relative weight of a particular wavenumber k_1 in the collection is the spatial finite-volume Fourier-transform $\delta\mathcal{N}(t,k_1)$ of the fluctuation field $\delta N(t,\mathbf{r})$ computed over the volume. The spectrum $\mathcal{E}(\omega)$ thus corresponds to the temporal Fourier transform, $\delta\mathcal{N}(\omega,\mathbf{k})$, of $\delta\mathcal{N}(t,\mathbf{k})$. When incoherently summed over whole illuminated volume, the $\delta\mathcal{N}(\omega,\mathbf{k})$ becomes the overall plasma fluctuation power spectrum, the random variable central in the IS theory.

[5]Paul M. Bellan: Fundamentals of Plasma Physics, 2008, Cambridge University Press.

Discussion

The double-bump IS ion-line spectrum is due to scattering from a large number of short-duration, short-extend, wave-length matched, ion-acoustic density waves, both upgoing waves and downcoming waves. The bumps are concentrated on the frequencies that are shifted, down and up, from the radar frequency by the frequency of the density wave. Wavelength-matching density fluctuation is required to get strong constructive interference at reception, but, for the rather short individual waves, the matching needs not be very precise, and so a whole spectrum of wavelengths around the exactly-matching wavelength can contribute significantly to the received field.

To get enhanced scattering only requires a matching fluctuation; from that point of view, the fluctuation could as well be frozen. But if the fluctuation is actually a wave, a frequency shift result. The main point of this note has been to stress that the mechanism of the frequency shift is not the same as that of the normal Doppler-shift. In the normal Doppler-shift, the scatterers, the individual electrons, experience different acceleration when the electrons are moving with respect to the radar than when they are not. This is not the case when the scattering is due to a density wave. From the point of view of the primary scatterers, a density wave is not equivalent to a beam of particles moving with the phase velocity.

A density wave is not even equivalent to the frozen fluctuation starting to move, as a "block", with the phase velocity. In a sinusoidal density wave, the mean (thermal motion averaged out) motion of the scatterers is longitudinal harmonic oscillation with the wave frequency, and with velocities that per see have nothing to do with the phase velocity of the wave. The frequency shift by the wave frequency obviously is not due to this harmonic motion, which averages to zero.

Even though the scattering "due to the wave" ultimately is due to the scattering by the essentially stationary electrons, it is best to forget about the individual scatterers and zoom out to view the plasma as a fluid, and describe the situation in terms of the density fluctuation. Then, we could model the scattering of the radar wave from a density wave like this: When a density wave moves through the a few wavelengths wide scattering region, the small sub-volumes act like amplitude modulators, the amplitude just representing the number of electrons in that volume. The modulators are initially phased to produce a frozen sinusoidal fluctuation through the scattering region, and then the density at each sub-volume starts varying harmonically. When the scattering region is illuminated by the radar wave, each sub-volume acts like a amplitude-modulated tiny transmitter. Due to the initial phasing of the transmitters, either an upgoing or down-going pattern of transmitter phases results, depending in which direction (+$\omega_{\rm w}$ or -$\omega_{\rm w}$) the harmonic oscillation occurs. The received signal is the summed-up signal from these transmitters. This model is a direct hardware realization of the superposition integral with a density of type $N_{\pm}(t, z) = 1 + A\cos(Kz \pm \omega_{\rm w} t)$.

The modulator spectrum has a DC component and sidebands at $\pm\omega_{\rm w}$. This spectrum is then shifted (mixed) by the radar carrier frequency $\omega_{\rm rad}$. But we have seen that when the transmitted waves from the elementary trasmitters interfere at reception, for the upward going density wave pattern the "Bragg-condition" strongly favours $\omega = \omega_{\rm rad} - \omega_{\rm w}$, suppressing the carrier (DC-component) and the $\omega_{\rm rad} + \omega_{\rm w}$ component.

In this picture of the scattering there is no place for the electrons themselves emitting any Doppler-shifted radiation (apart from the plasma bulk Doppler-shift). The fact that the frequency shift $\omega_{\rm w}$ nevertheless numerically equals the Doppler-frequency shift computed by assuming that a wavefront of the density wave behaves like a beam of particles moving with the wave phase velocity, may seem almost coincidental, but of course, is not.

What happens is that the net scattering comes from the sub-volumes were most of the scatterers are, that is, from a density wave crests. For the sake of simplicity, we may imagine that those are the only places that radiate at all. At any moment of reception, the scattering is so arranged that radiation from all the crests add with equal phase at reception. So consider some particular crest as if it would be the only scatterer. The phase at reception resulting from scattering through that volume can depend only on the total phase path length from the radar transmitter to the scattering volume and back. The crest acts as kind of marker for the volume where most of the scattering is coming. When the crest moves, even though the scattering particles don't move, with the phase velocity of the wave, the phase path through the contributing scattering region changes, precisely in the same way as when a bunch of particles were moving with the crest. We have shown that the rate of change of the phase path length determines the size of the frequency shift for the scattered field, so it is clear that the frequency shift value must be the same in both cases.

I acknowledge that sensible people can disagree with me on this, but I maintain that just to be able to calculate the size of the frequency shift from the phase path change is not the whole story. In case of the normal Doppler-shift, descriptions P1–2 on p.2 clearly provide additional insight into the phenomenon. In case of scattering from the wave, there is less physics to explain. Perhaps the main thing to explain is that there really is not much more to explain. The primary scattering from the wave is the same kind scattering from the electrons as with the normal Doppler-shift, the scattering now having a Doppler-shift zero.

The observed frequency shift arises due to target-switching, not due to target motion. I have suggested proposition P3 (p.5) as a way to summarize the effect. The key observation is that the overall phase of the fluctuation—the locations of wave crests—determines, or selects, from which sub-volumes the dominant, "phase-determining", contributions to the received field come at a given moment of reception. The range to the volume has significance, because it determines the overall phase delay of the reception with respect to transmission. When the contributing range, say, becomes larger, the received signal inherits its phase from relatively further and further back in time, as shown in Fig. 8. This introduces on extra change of phase, at the rate of the density wave's angular frequency, on top of the baseline change that occurs simply because the transmitted phase changes. That extra change is responsible for the frequency shift associated with the density wave.

Of course, with the wave-organized, semi-coherent scattering, calculating the frequency shift is not the main thing. The interesting issue is explaining the nature of the waves themselves, in various physical circumstances, and how this shows up in the detailed shape of the IS spectrum. That is were the actual physics of IS is.

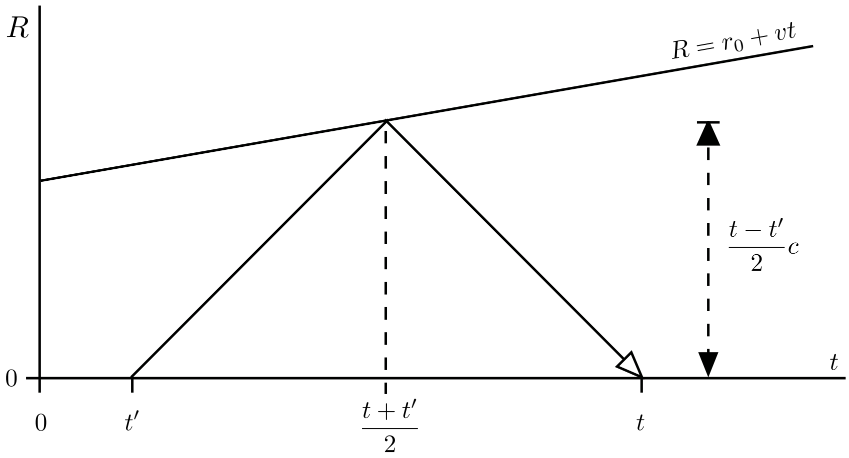

Figure 1: The delayed time. The space-time diagram shows the world-line of an scattering object moving away from the radar with velocity v, and the world-line of some specific phase front of the electric field. The delayed time t' is solved from the equation $\frac{t-t'}{2}c = r_0 + v\,\frac{t+t'}{2}$.

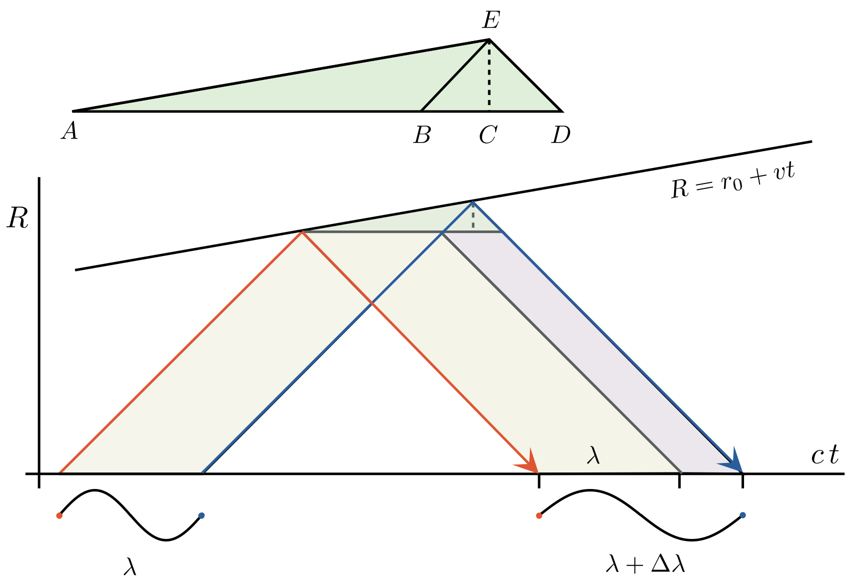

Figure 2: Doppler-shift. The space-time diagram shows an electron moving at velocity v away from the radar, and two radar wavefronts, 360° apart in phase, reflected from it. The Doppler-shift $\Delta\lambda$ can be solved purely from the geometry, using triangles in the green-coloured area of which an enlarged and annotated copy is shown at the top of the diagram. We use unit such that $c = 1$. Then there are enough 45° angles, 90° angles, and parallelograms around, for the following to hold: $AB = \lambda$, $BD = CD = \Delta\lambda/2$, $CE = \lambda/2$. The velocity is $v = \Delta R/\Delta t = CE/AC$, so that $v/c = (\Delta\lambda/2)/(\lambda + \Delta\lambda/2)$. Solving this for $\Delta\lambda$ gives $\Delta\lambda = 2\lambda\,(v/c)/(1-v/c) = 2\lambda\,v/(c-v)$. In terms of frequency $\omega = 2\pi c/\lambda$, this becomes $\omega_{\mathrm{D}} = (c - v)/(c + v)\omega$, in consistency with Eq. (4) and Eq. (7).

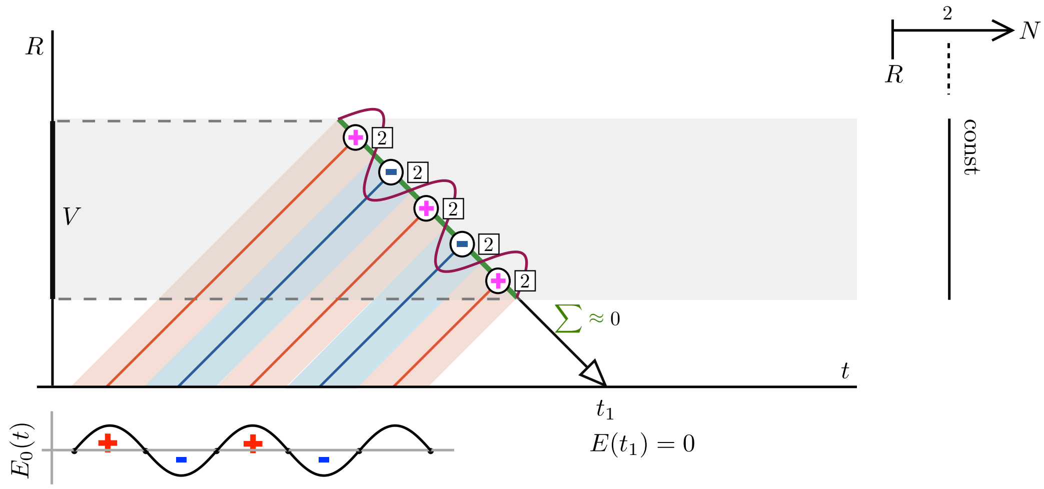

Figure 3: Constant density does not give any net scatter. The figure shows world-lines of constant phase associated with the scattering events that contribute to the received field E at the time t_1. In the continuous-density picture, we image the target divided into very small volumes δV_n within the scattering volume V (which itself also is assumed small), and in each δV, we imagine one scattering event. The up-going legs of the world-lines corresponding to the crests and throughs of the outgoing wave are shown with red and blue lines respectively, the remaining world-lines are indicated by the coloured zones. The scattering events contributing to $E(t_1)$, at the vertex of their world-lines, are located on the green-coloured area of the space-time diagram. A scattering event n gives a contribution δE_n to the received field that is proportional to the number of electrons in the volume δV_n (the number inside the small square), times the phase factor $\mathrm{e}^{i\phi_n}$ that the volume picks from its associated world line (circled plusses and minunes, and more generally, the pink sinusoid drawn along the scattering region). The received field $E(t_1)$ is the sum of the contributions δE_n. With constant density (of value "2" in the illustration), the contributions δE_n are essentially just the phase factors $\mathrm{e}^{i\phi_n}$, and these sum up to zero, apart from edge effects within a single wavelength. In the diagram, there actually is a net scattering of size "2". In general, the wave coherence volume V may be small enough so that the edge effects couldn't be totally ignored, but we will nevertheless do so; the whole discussion is highly non-quantitative anyway.

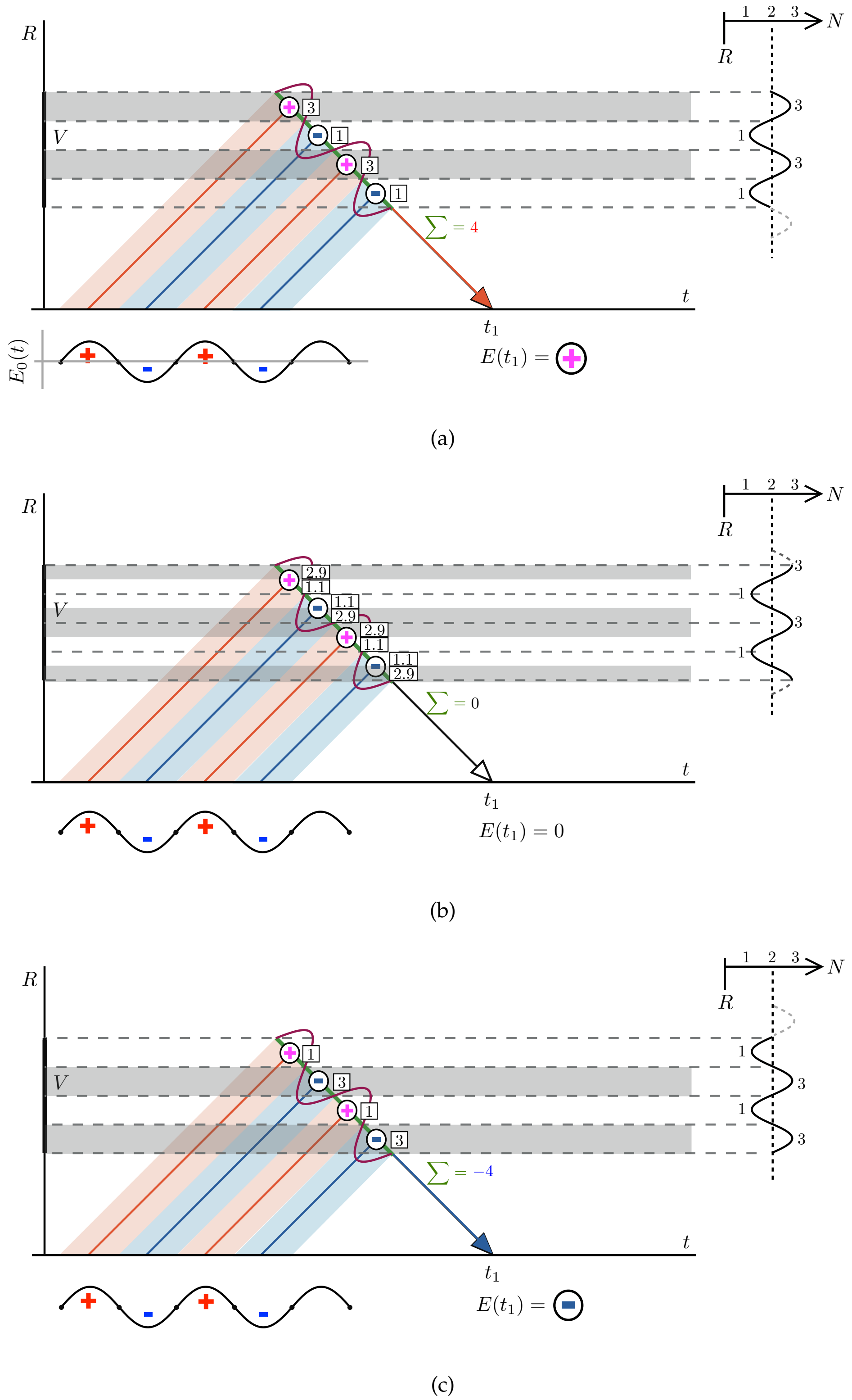

Figure 4: Effect of the initial phase of a frozen fluctuation. The panels are identical apart from the phase of the fluctuation, which changes by $\pi/2$ from (a) to (b) and again from (b) to (c). The phase of the received field changes precisely by the same amount.

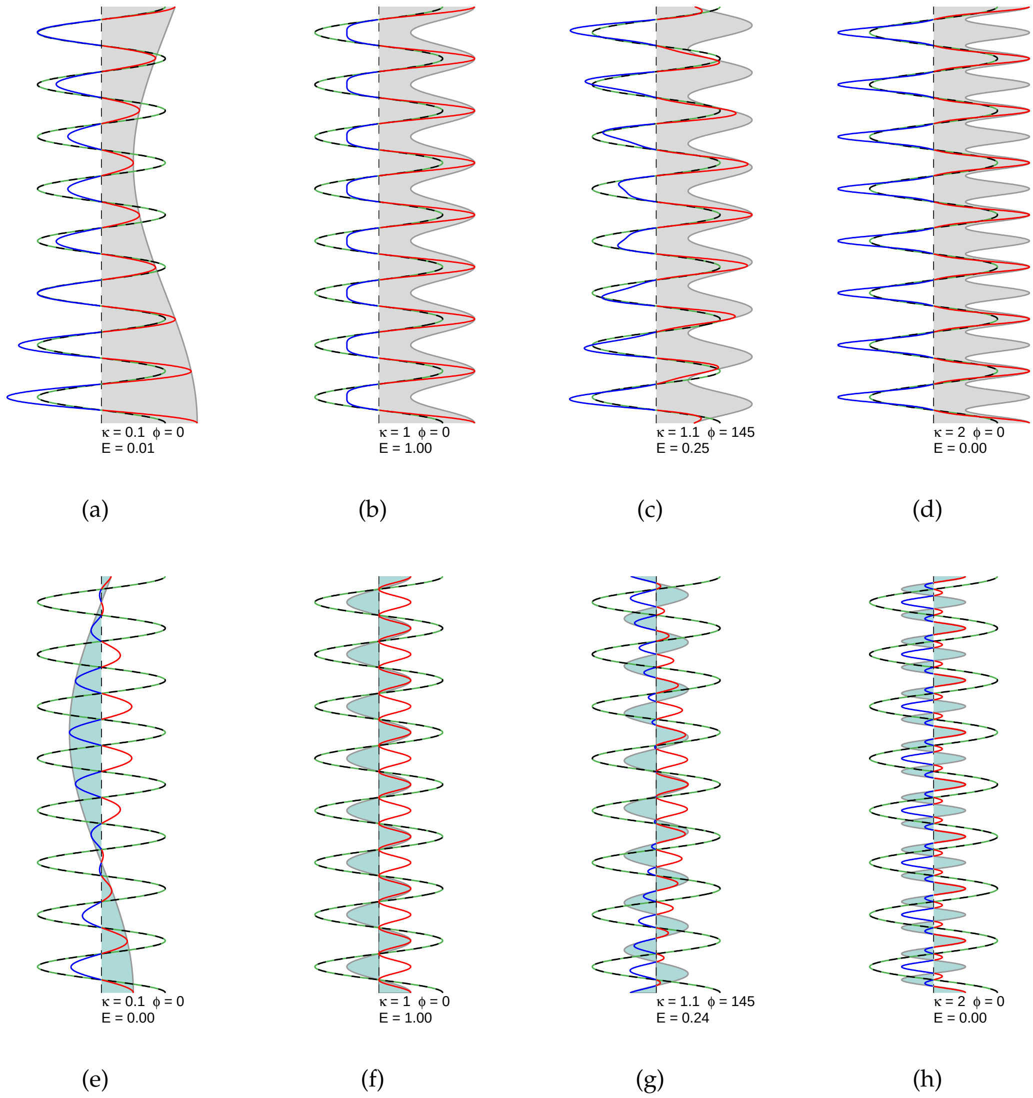

Figure 5: Matching and non-matching, frozen, fluctuations. The figure shows the integrand of the superposition integrand $I = \int_L N \cos(-Kz) dz$, for different fluctuation wavenumbers k_{w}. In the top row of panels, the density $N = 1 + 0.5 \cos(\phi - k_{\mathrm{w}} z)$ is shown as gray-coloured surface, the phase of the transmission $\cos(-Kz)$ as green-black dashed line, and their product as blue(=negative) and red(=positive) line. In the bottom row of panels, the fluctuation $\delta N = 0.5 \cos(\phi - k_{\mathrm{w}} z)$ is used instead of the density N itself. The quantity κ is the ratio of the fluctuation wavenumber to the ideally-matching wavenumber $K = 2k_{\mathrm{rad}}$. The value E is the value of the superposition integral divided by the ideally-matched value, $E = I(k_{\mathrm{w}})/I(K)$. The range interval L is $4 \times \lambda_{\mathrm{rad}}$ in all panels.

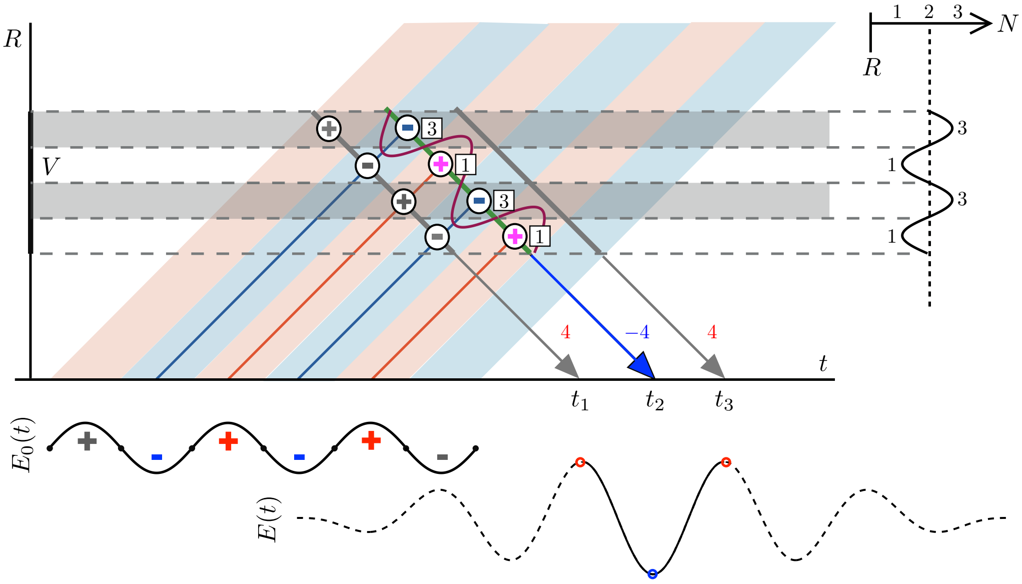

Figure 6: Scatter from a frozen fluctuation. Construction of the signal for the time t_1 was shown in detail in Fig. 4.a; this figure shows the construction at the time t_2, half a period later. At all contributing spacetime points, the phase of the radar field has progressed by π between t_1 and t_2. Therefore, the phase of the received field has changed by the same amount, and so the reception has changed from "+4" to "-4". In general, the phase of reception tracks the phase of transmission without any frequency shift. The only effect of the fluctuation is to enhance signal strength. To compute the shape of reception $E(t)$ in detail, it is convenient to view $E(t)$ as the output of a filtering operation where the filter is the fluctuation and the transmission $E_0(t)$ is the signal. In this example, the transmission is 3 periods long, and the fluctuation 2 periods long. Only one period of undistorted, maximal amplitude signal is received, from t_1 to t_3; the rest of the 5 periods long reception has reduced amplitude.

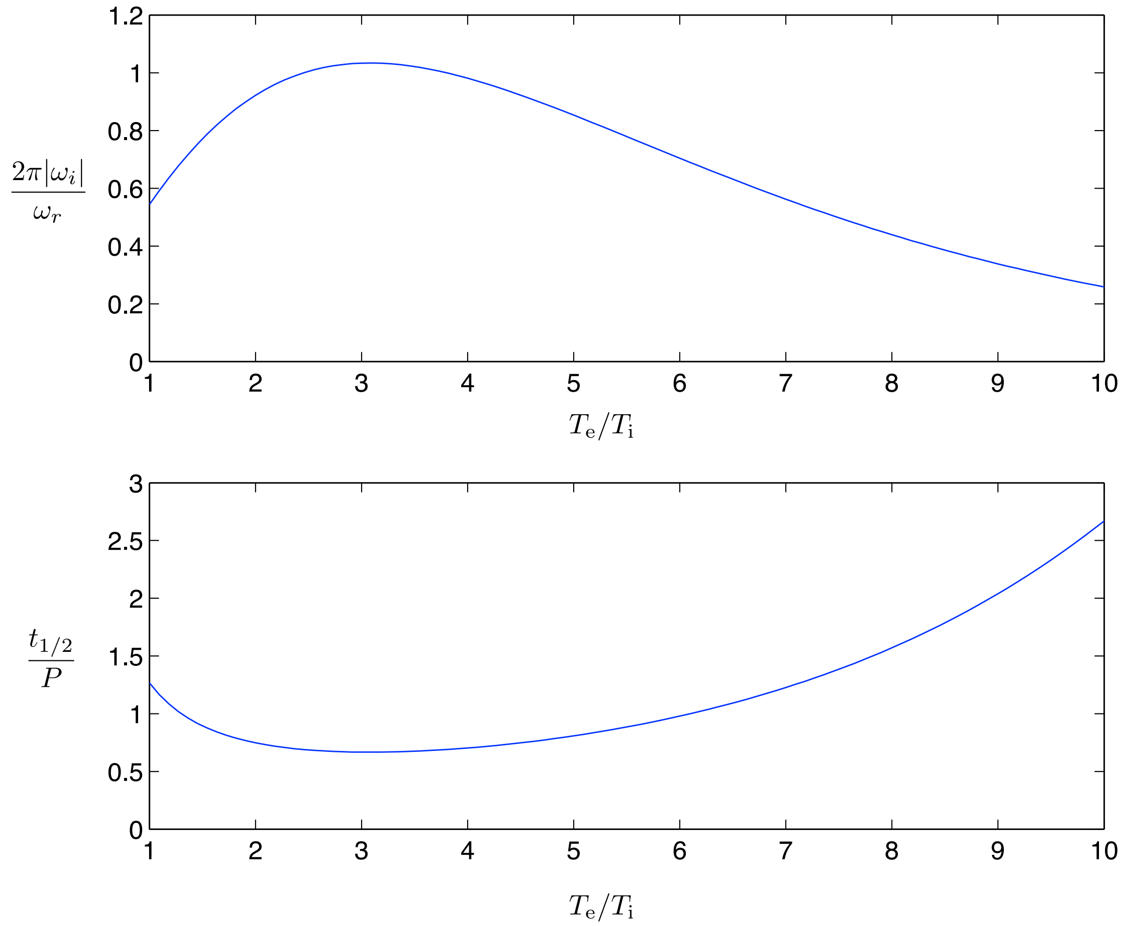

Figure 7: Damping of ion-accoustic waves. The top panel shows the attenuation coefficient $a = 2\pi\frac{|\omega_{\rm im}|}{\omega_{\rm re}}$ in $\delta N(t,z) \sim {\rm e}^{-a\frac{t}{P}} \times {\rm e}^{i(\omega_{\rm w}t - Kz)}$ of a Landau-damped ion-accoustic wave, as a function of $T_{\rm e}/T_{\rm i}$, for typical ionospheric F-peak parameters at EISCAT UHF. The bottom panel shows the decay half-time $t_{1/2}$ of the wave amplitude in terms of the wave period P, as function of $T_{\rm e}/T_{\rm i}$. In typical ionospheric condition, the ion-acoustic wave propagates only of the order of one wavelength.

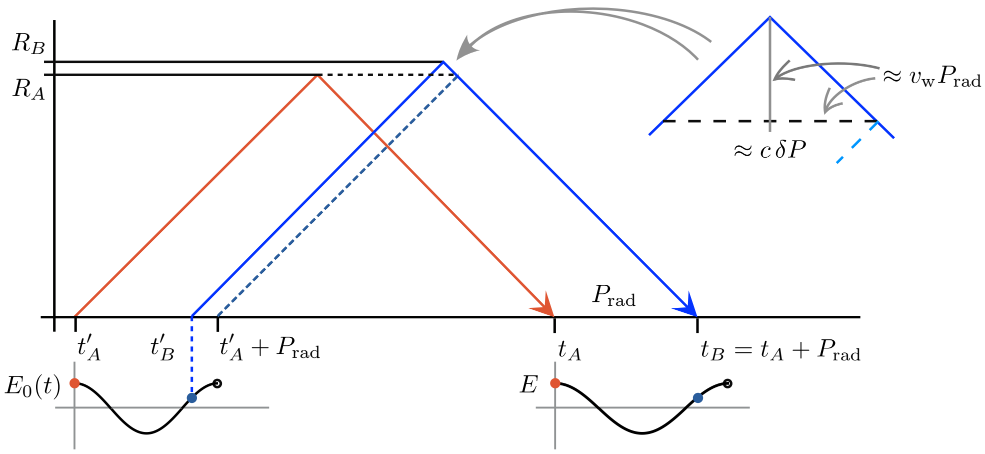

Figure 8: Frequency shift due to looking progressively further back in time. We consider some particular wave-crest of an outgoing density wave. The position of the crest tells from where the dominant, phase-determining, contribution to the reception comes. We consider two reflections of radar signal from layers of stationary electrons, at ranges R_A and R_B, marked by that wave-crest. The first reflection is received at time t_A, from range R_A. The signal has the same phase as the transmitted wave had at the time $t'_A = t_A - 2R_A/c$. We have here chosen that phase to be zero (the "red" phase). We have chosen to take-in the second signal at time t_B, precisely one period, P_{rad} of the transmitted wave, later. If the density fluctuation would be frozen, this signal would again be coming from range R_A, and would have started its journey at $t'_A + P_{rad}$ (the dashed line). With the traveling wave, the signal is still coming from the position of the density wave crest, but now from a larger range, $R_B \approx R_A + v_w\, P_{rad}$. Because of the longer pulse flight time, the signal we receive at t_B inherits its phase from the time $t'_B = t'_A - \delta P$, the time of the "blue" phase. The diagram indicates that at t'_B, hence at t_B, the first full period is not yet completed, so the period at reception is longer than the period of transmission. It follows from the geometry of the diagram that, for $v_w \ll c$, the change δP of the signal period between transmission and reception is, as expected, $\delta P \approx (2v_w/c)P_{rad}$. We note that the overall situation here is quite different from the case of normal Doppler-shift, where we would have a single layer moving from R_A to R_B. Due to their motion, the electrons in that layer would actively change the characteristic of the radiation, causing the Doppler-shift. In the present case, radiation coming from layers R_A and R_B separately, has zero Doppler-shift, and the perceived frequency shift is due to the observer seeing a different stationary target as the time goes on.