Experiment design: CODING

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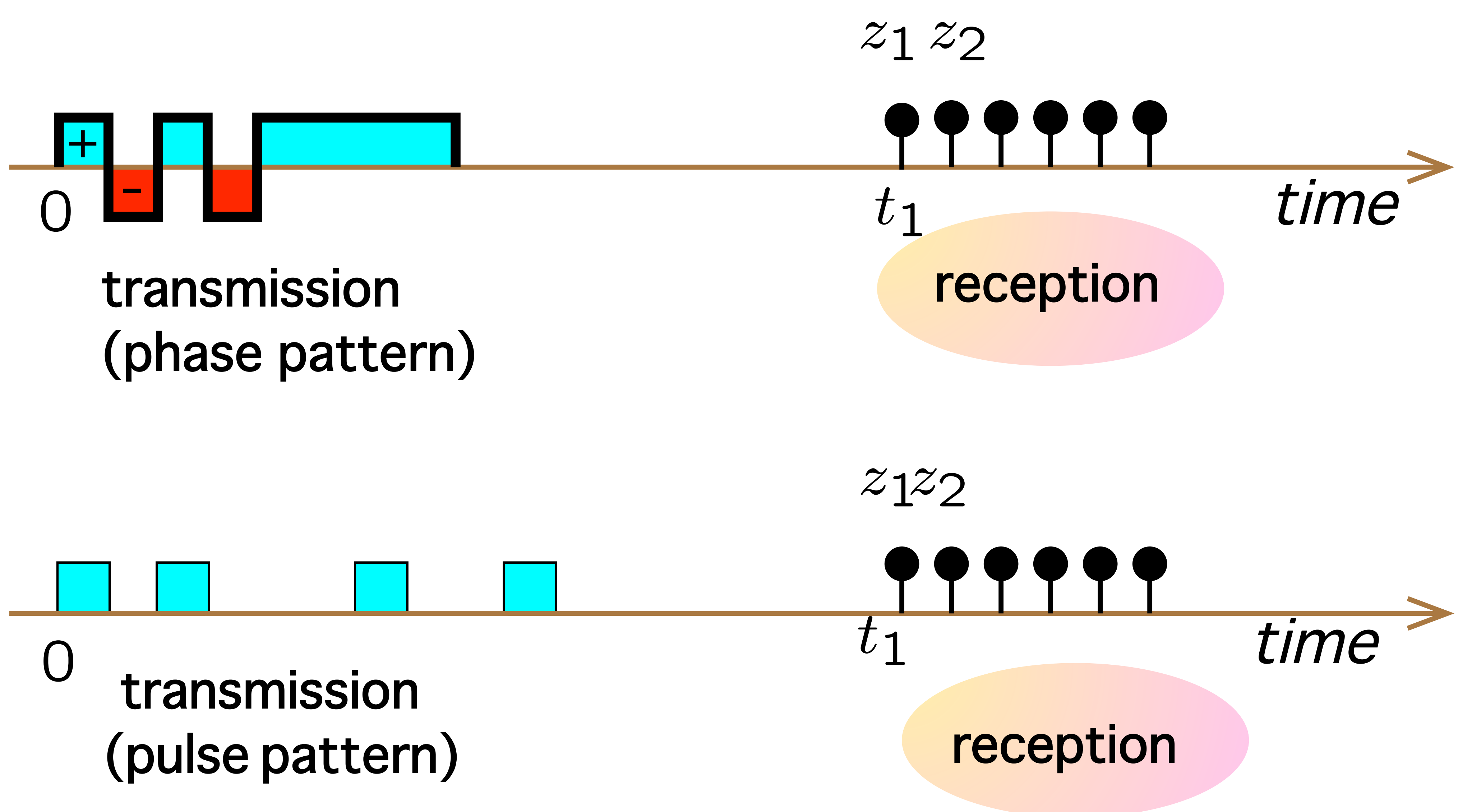
An almost "free lunch":

Improving
the range resolution
of the target spectral estimates
$\mathcal{S}(\omega; R, t)$
without paying much for it.

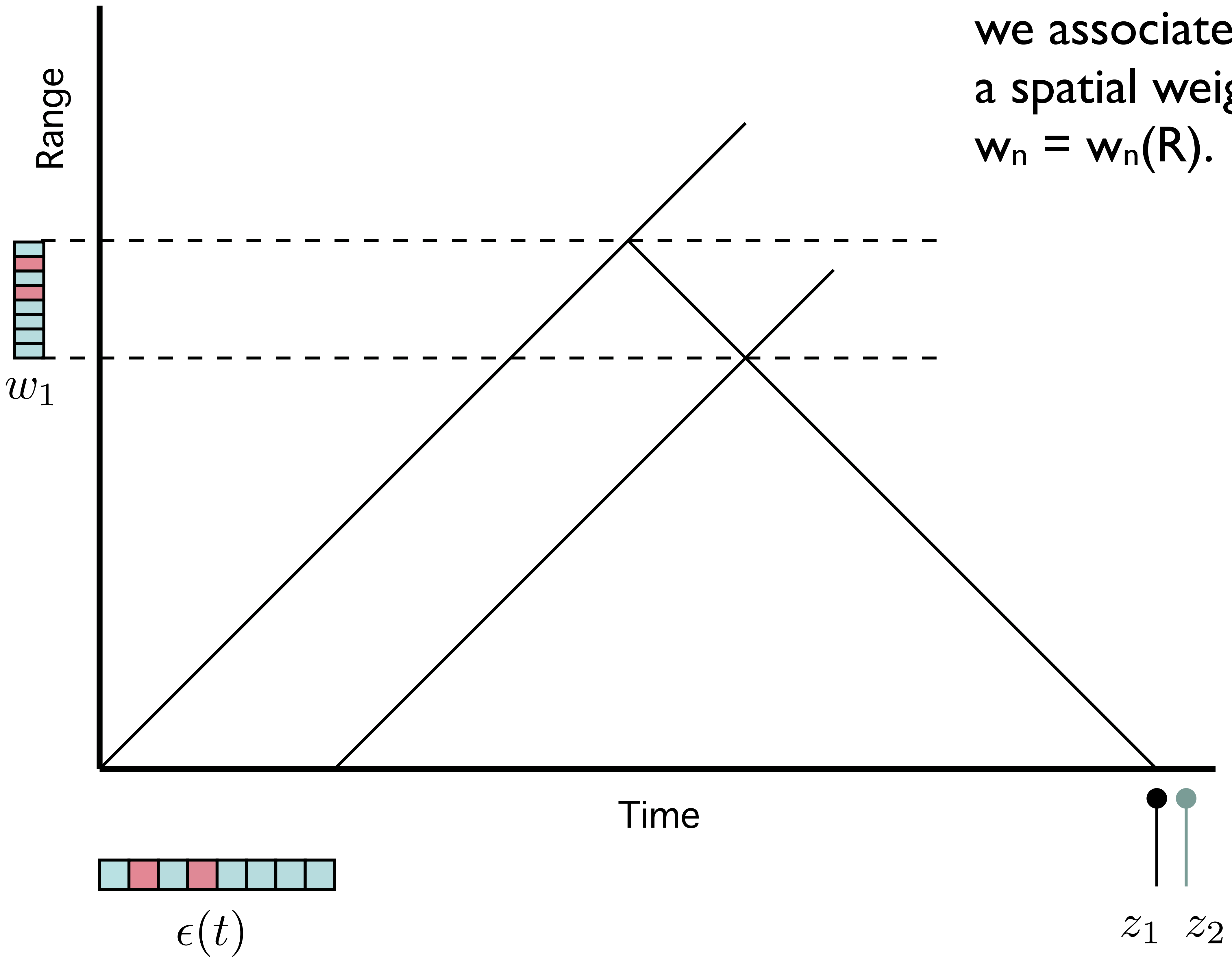
Coding possibilities (=ways to structure target illumination)

Transmission

Reception (complex samples)



Spatial weight of signal samples (1/2)



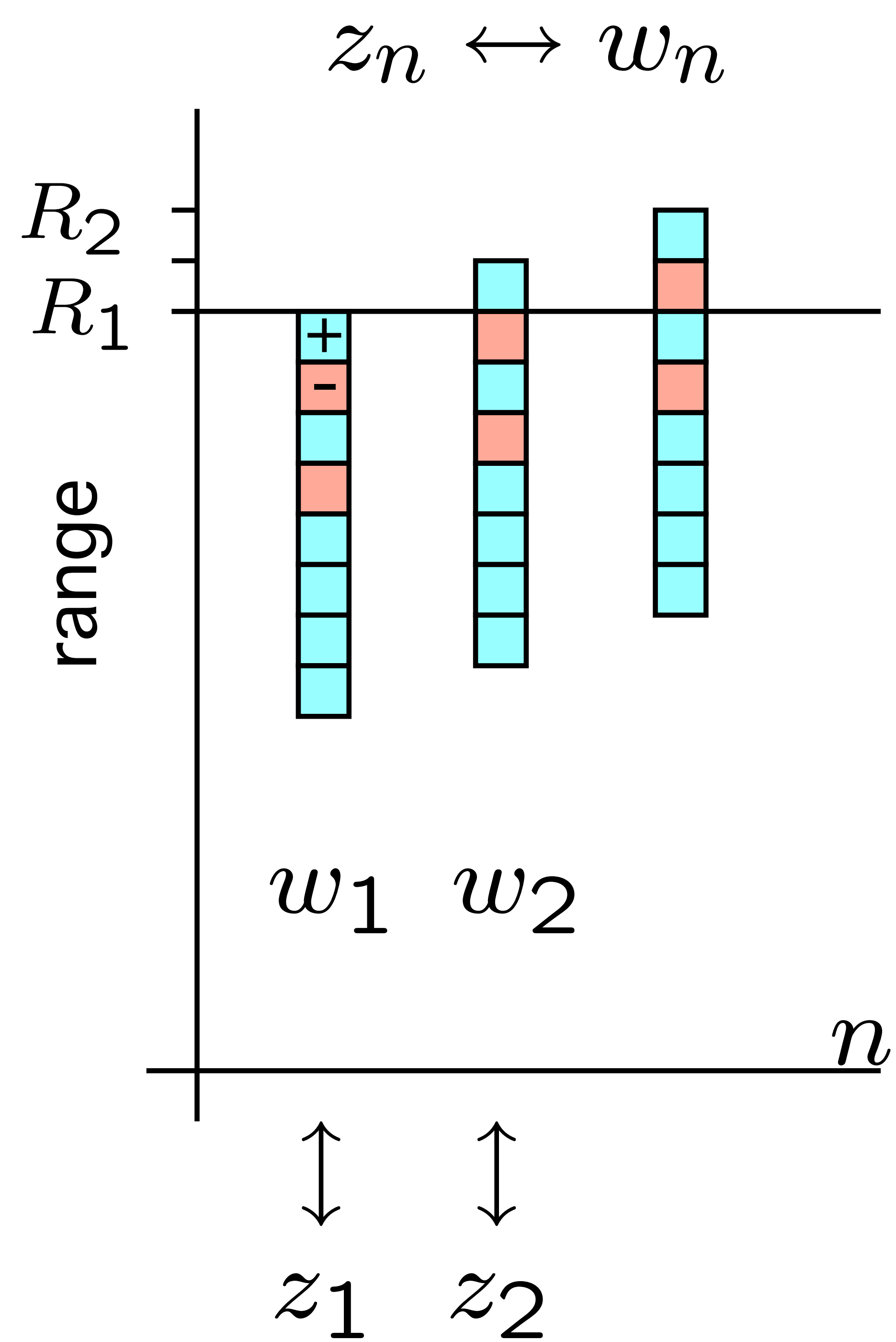
To each signal sample z_n,
we associate
a spatial weight function
$w_n = w_n(R)$.

Spatial weight of signal samples (2/2)

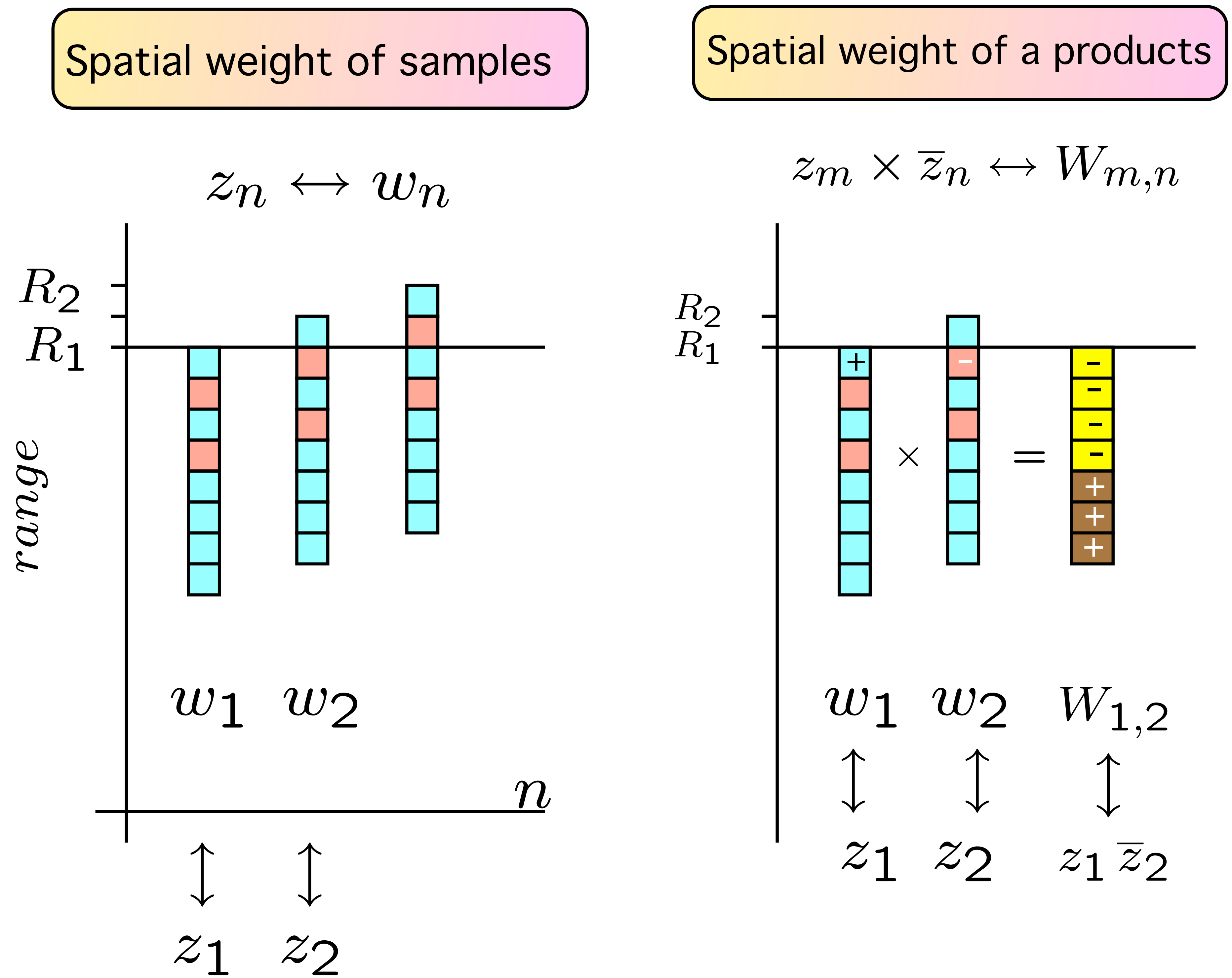
To each signal sample z_n,
we can associate
a spatial weight (function)
$w_n = w_n(R)$.

Here, we take the weight
to be just
the transmitted pattern.

In reality,
the weight is modified
somewhat by time-averaging
done in the receiver.



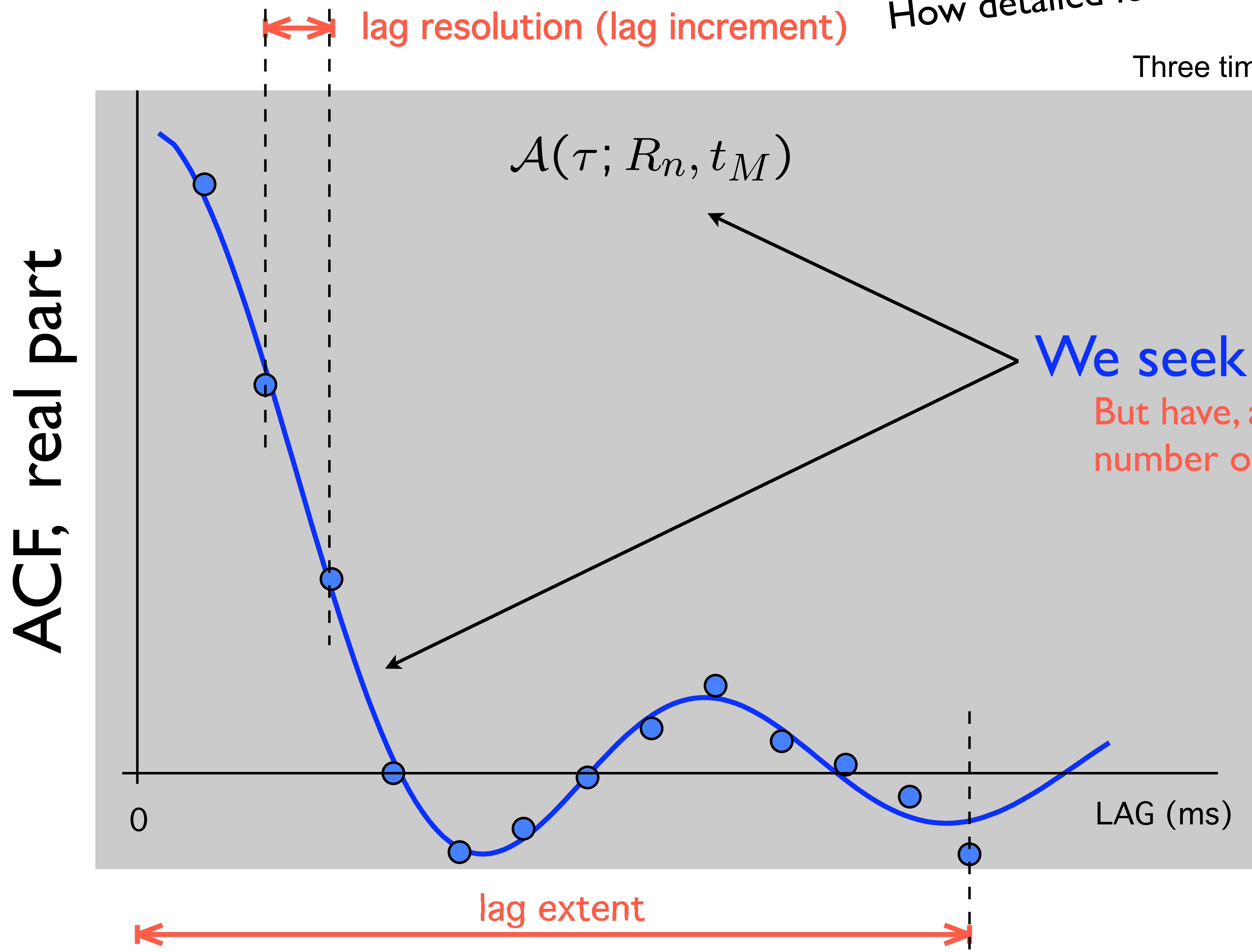
Spatial weight of ("raw") lagged products



Parameters related to ACF and SPEC measurement (1/3)

How detailed features can be "resolved"

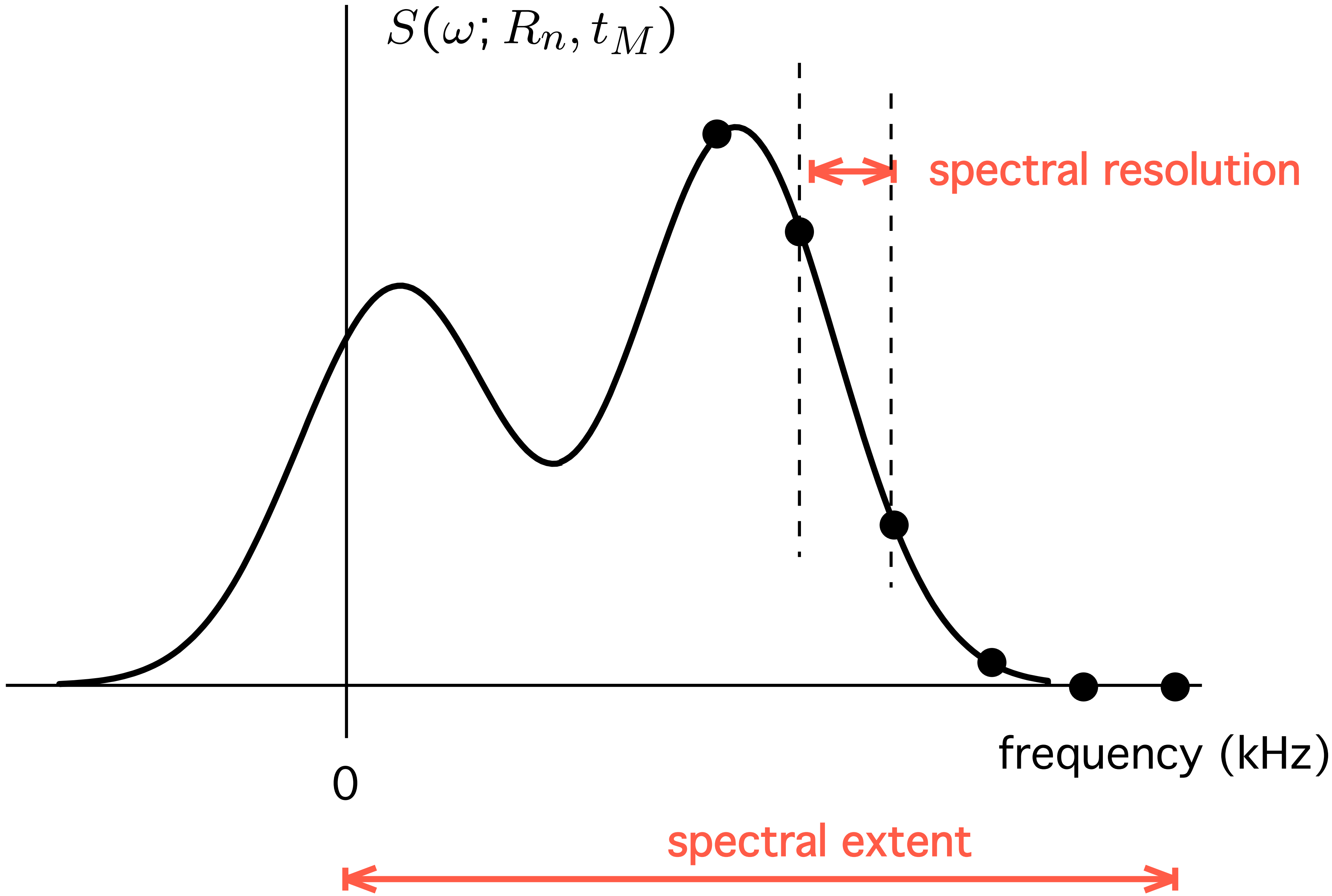
Three timescales: lag timescale
averaging timescale
time-of-day timescale



We seek to know this

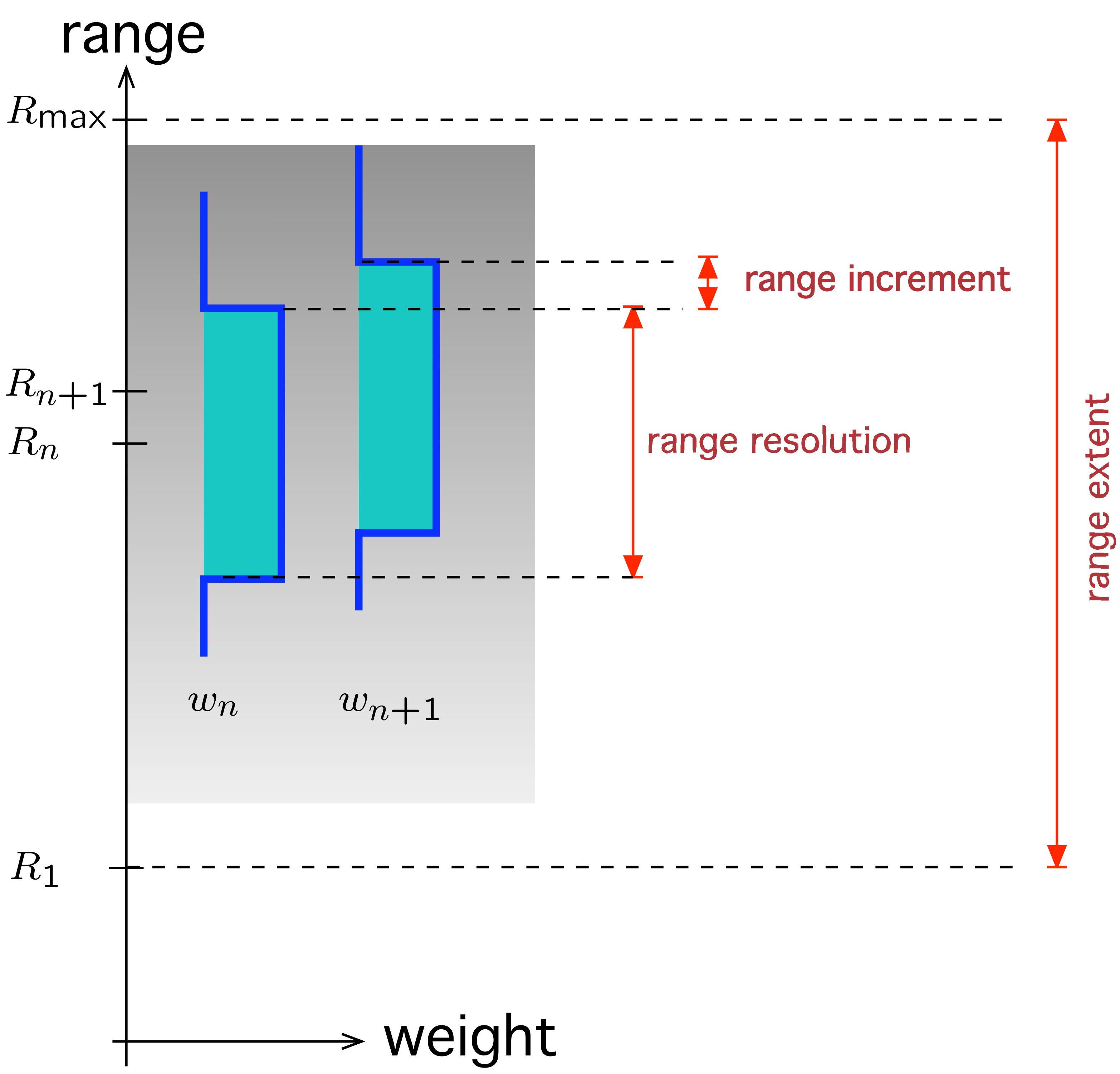
But have, at best, only a finite number of discrete samples

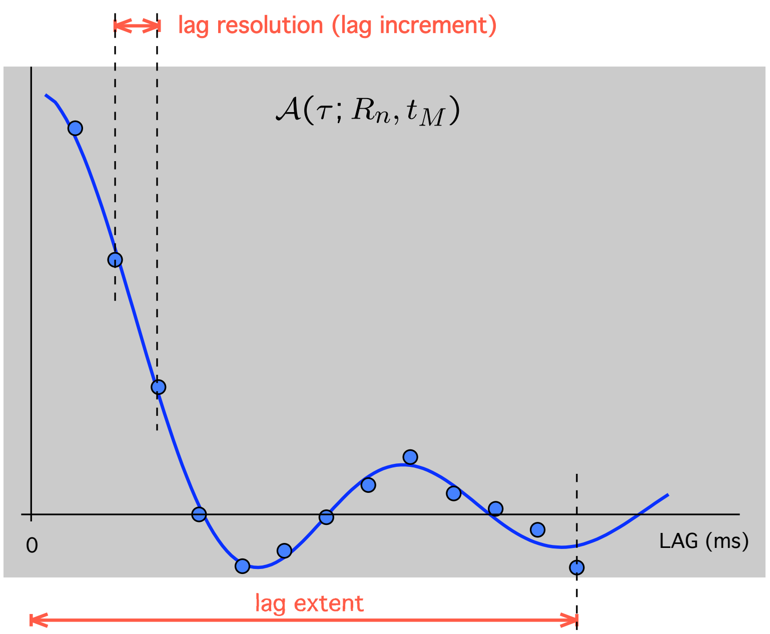
Parameters related to ACF and SPEC measurement (2/3)



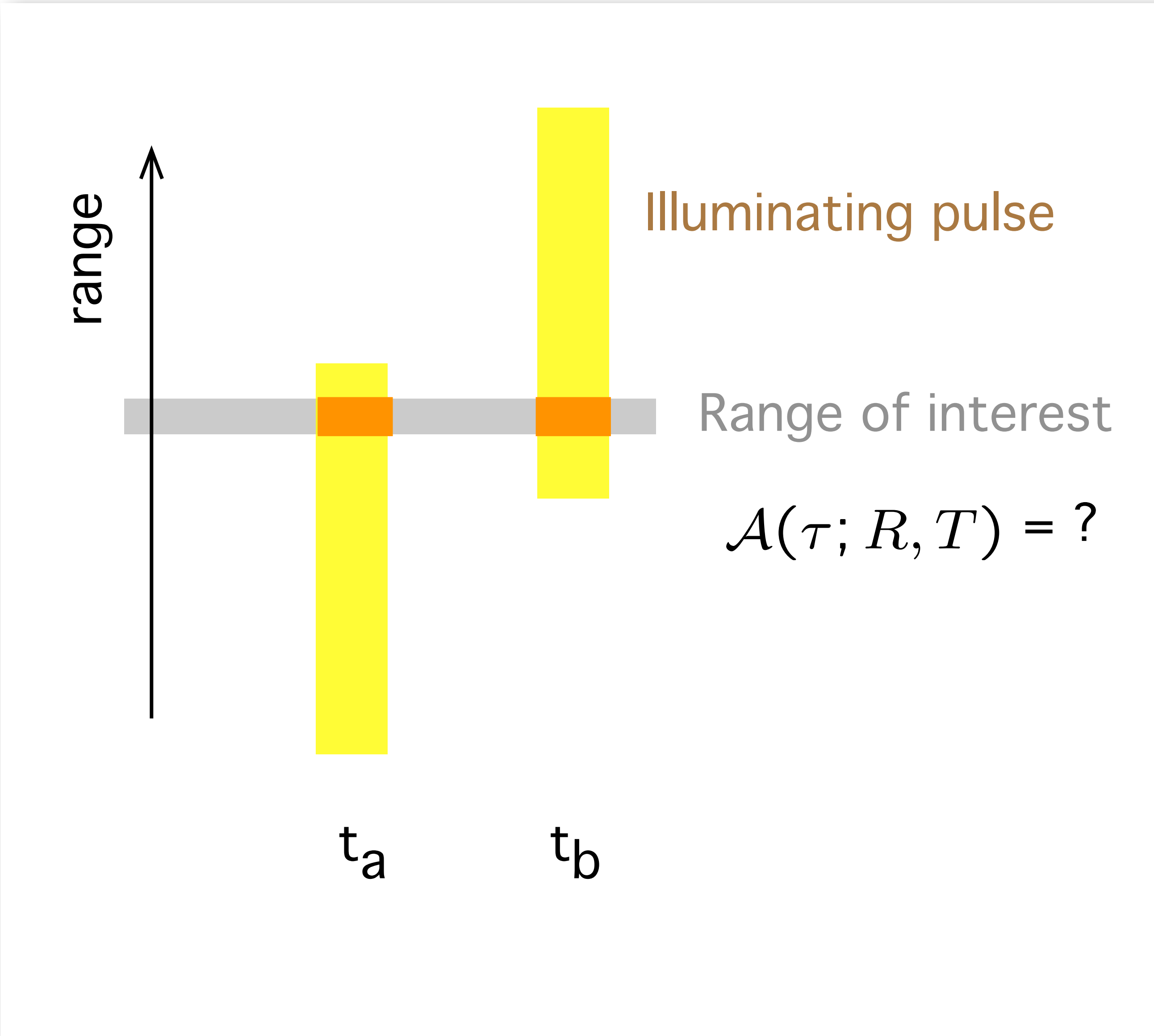
Parameters related to ACF and SPEC measurement (3/3)

$$\mathcal{S}(\omega; R, t)$$





Measuring the plasma ACF



$$\mathcal{A}(\tau) \propto E_t[\delta n_e(t; \mathbf{K})\overline{\delta n_e(t+\tau; \mathbf{K})}]$$

lag ❶ ; ❸

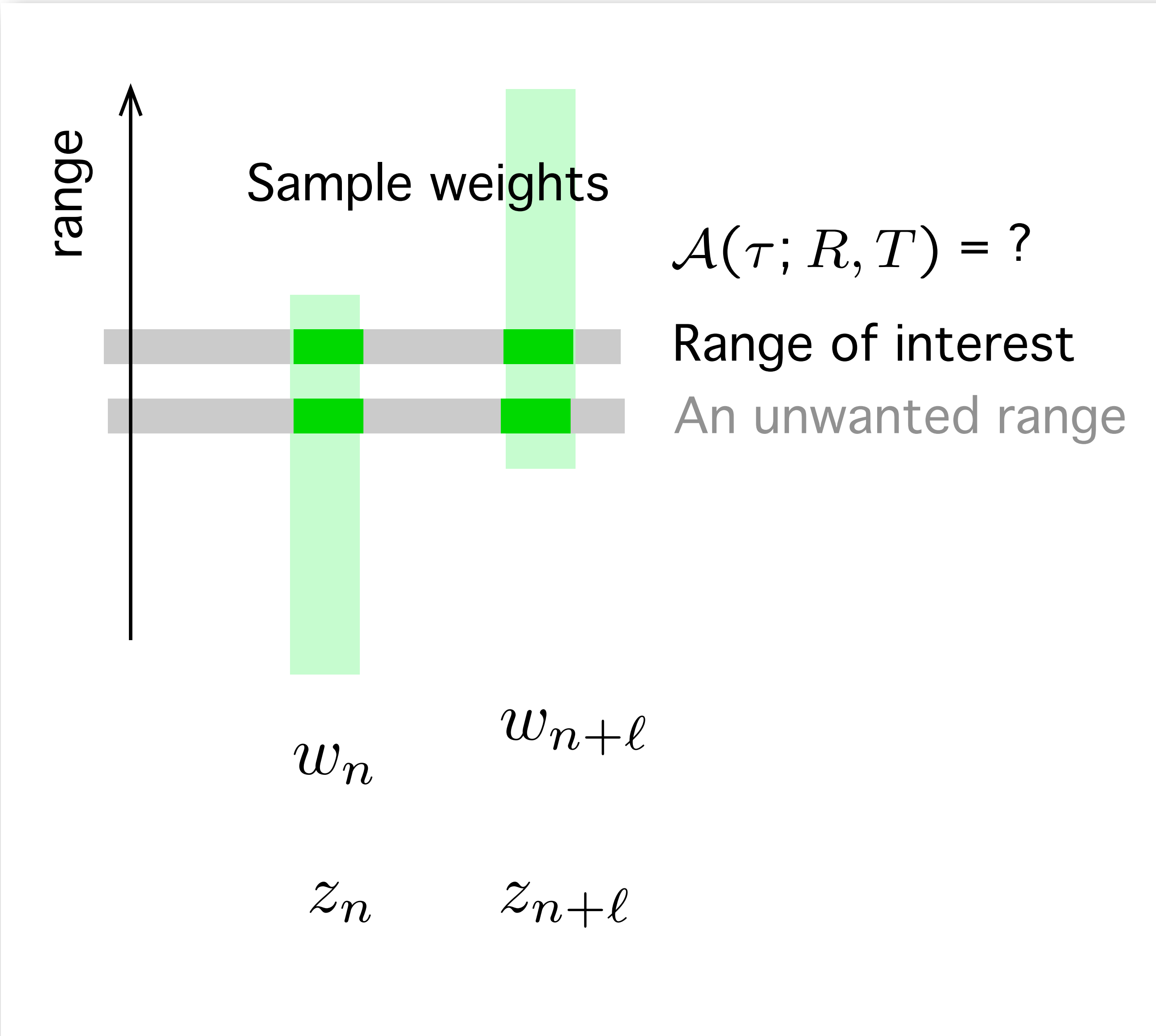
electrondensity fluctuation

❷ expectation value oper.

Three timescales: lag timescale ❶
averaging timescale ❷
time-of-day timescale ❸

Target must be illuminated long enough:
$t_b - t_a$ > Required lag extent

Measuring the plasma ACF using signal ACF



plasma ACF:

$$\mathcal{A}(\tau) \propto E_t[\delta n_e(t;\mathbf{K})\overline{\delta n_e(t+\tau;\mathbf{K})}]$$

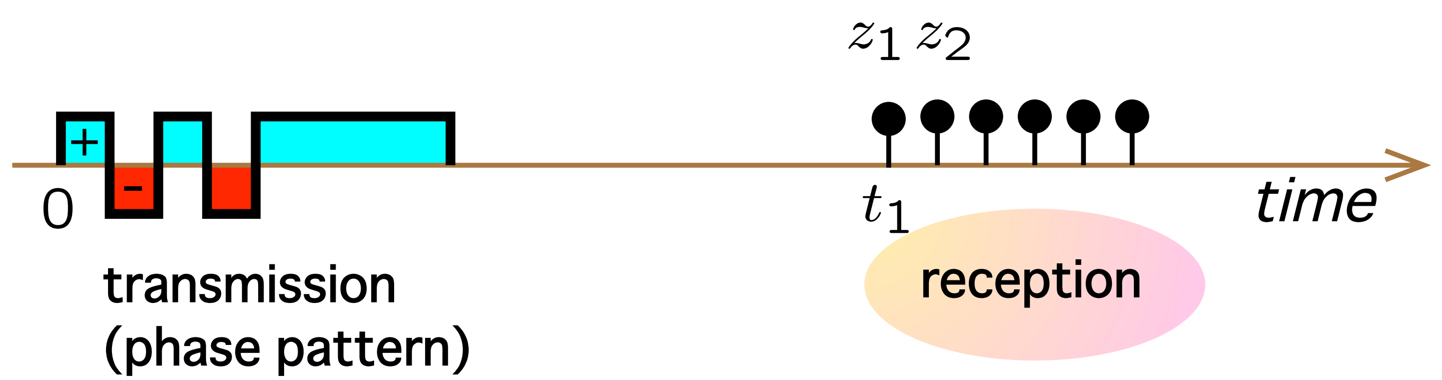
"raw" signal ACF:

$$A(\tau) = E[z_n \overline{z}_{n+\ell}]$$

The raw signal ACF is NOT a good estimate of the plasma ACF for any given range, due to contributions from unwanted ranges.

BUT, by coding the transmission, it is possible to cancel the unwanted contributions, with suitable processing ⇒"decoded" signal ACF.

An aside: LAG PROFILES (LP)



raw signal ACF for "range n"

$$L \mapsto A(L; n) = z_n \, \overline{z}_{n+L}$$

raw signal lag-L profile:

$$n \mapsto \mathrm{RLP}(L; n) = z_n \overline{z}_{n+L}$$

decoded signal lag-L profile:

$$n \mapsto \mathrm{LP}(L; n) \sim \sum_M a_M z_n^{(M)} \overline{z}_{n+L}^{(M)}$$

WANTED: accurate spectral measurements

with good spatial and temporal
resolution and extent and small steps

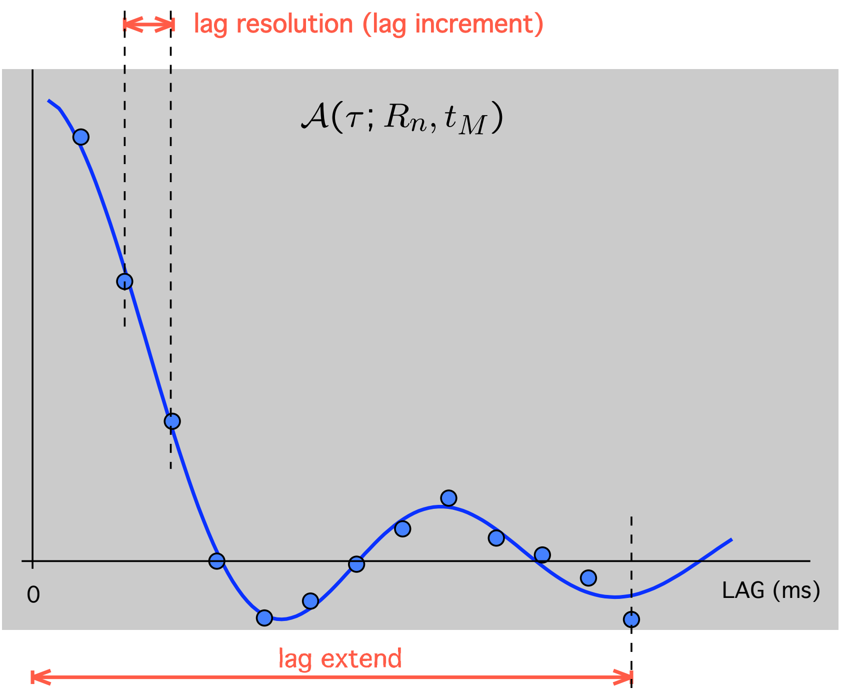
→ Good statistical accuracy
  High peek power (as high as possible)
  Long illumination (as long as possible)

→ Good lag resolution (large spectral extent)
→ Large lag extent (good spectral resolution)

→ Good spatial resolution
→ Large and unambiguos range extent

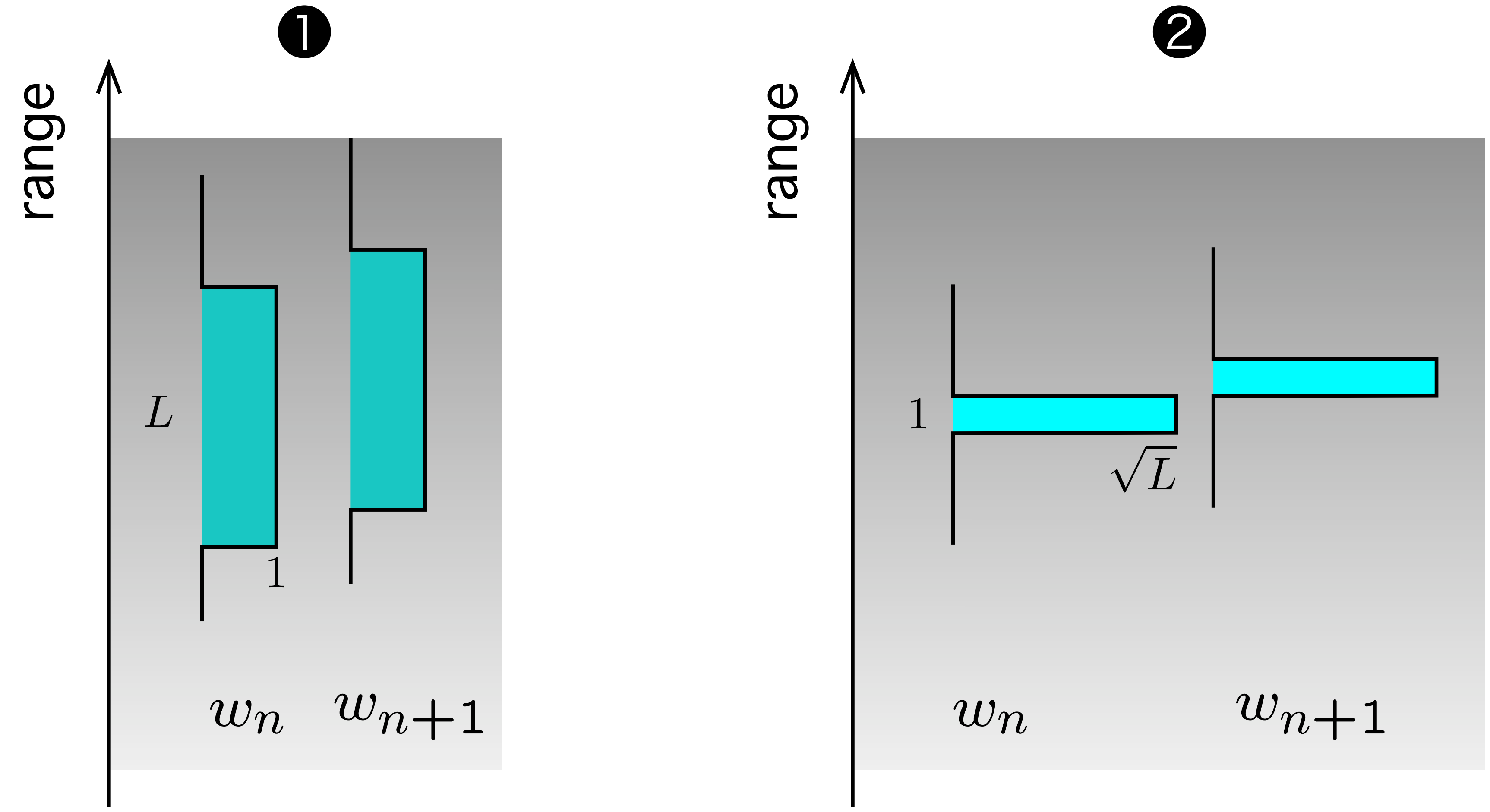
→ Good time resolution (short integration)

→ Large and unambiguous Doppler extent



WANTED: accurate spectral/acf measurements with good spatial and temporal resolution and extent, and small steps

Compare these



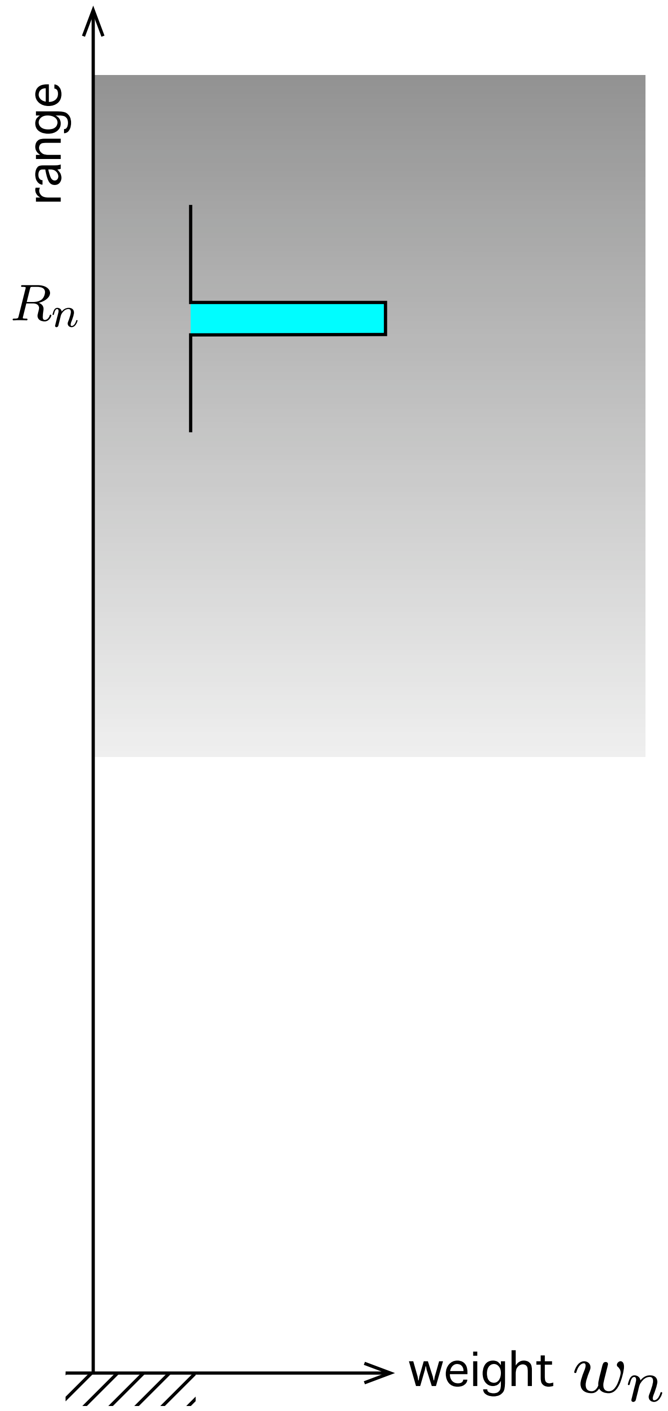
Bad range resolution

Good range resolution

Same range step
Same pulse energy
~Same stat. accuracy

Designer choices: Some not-so-good ideas

What about using a
single short but strong pulse ?



Designer choices: Some not-so-good ideas

What about using a single short but strong pulse ?

good range resolution

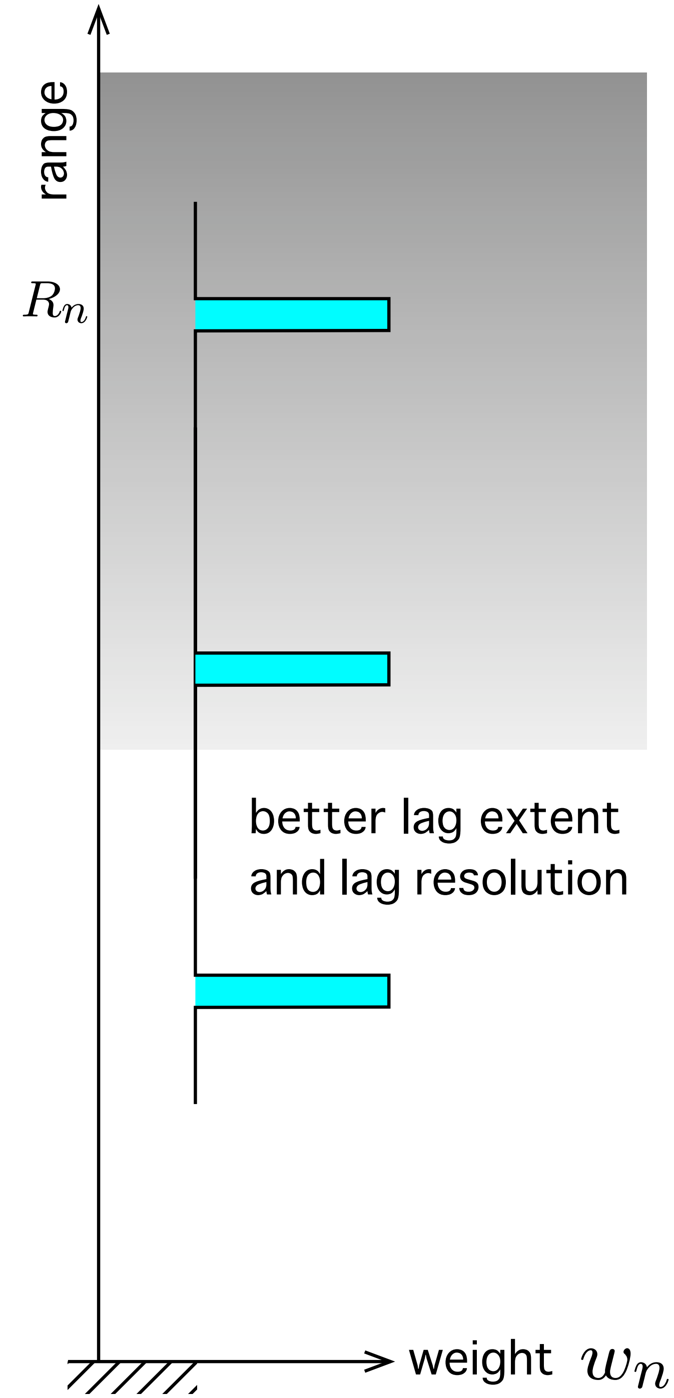
peak power limitation ?

bad lag extent !

Designer choices: Some not-so-good ideas

What about using a single short but strong pulse ?

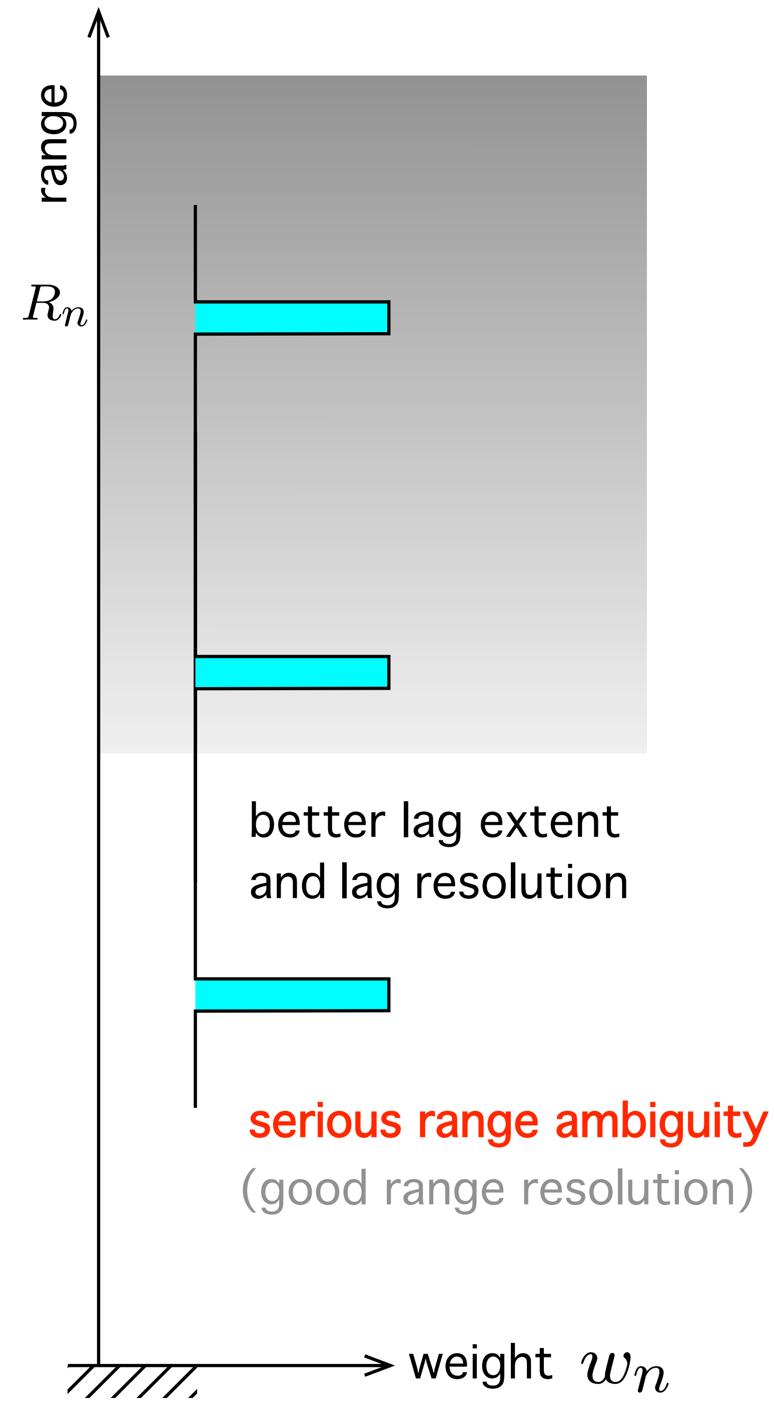
What about using a train of short but strong pulses ?



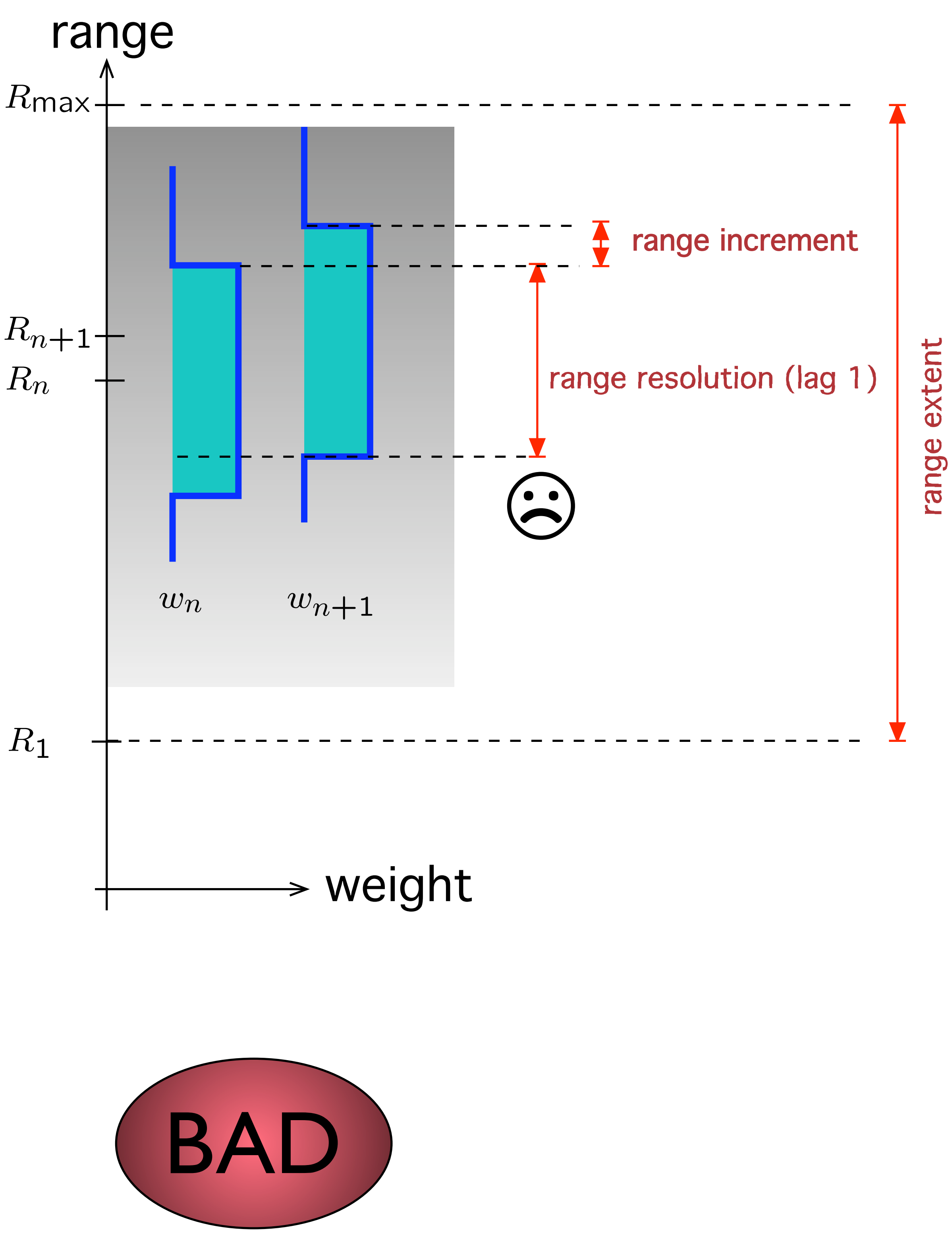
Designer choices: Some not-so-good ideas

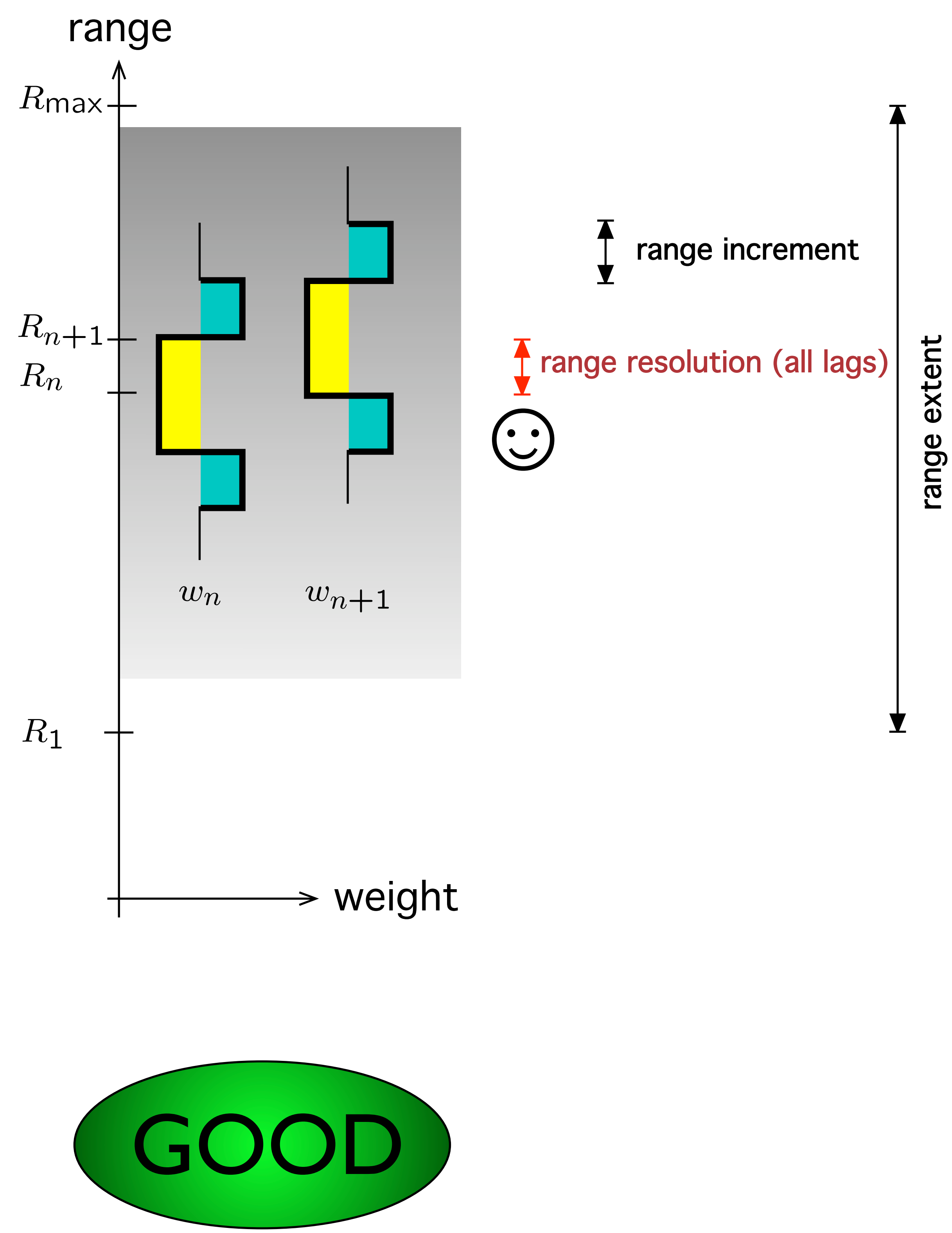
What about using a single short but strong pulse ?

What about using a train of short but strong pulses ?



A better idea: Long but structured illumination





Alternating codes by M.L., I.H., T.N., M.M.

By transmitting a cleverly selected set of binary phase codes,

and forming suitable linear combinations of the lagged products (sum over all the codes in the code set)

the spatial weight of the end result has length equal to the code bit length, with no sidelobes.

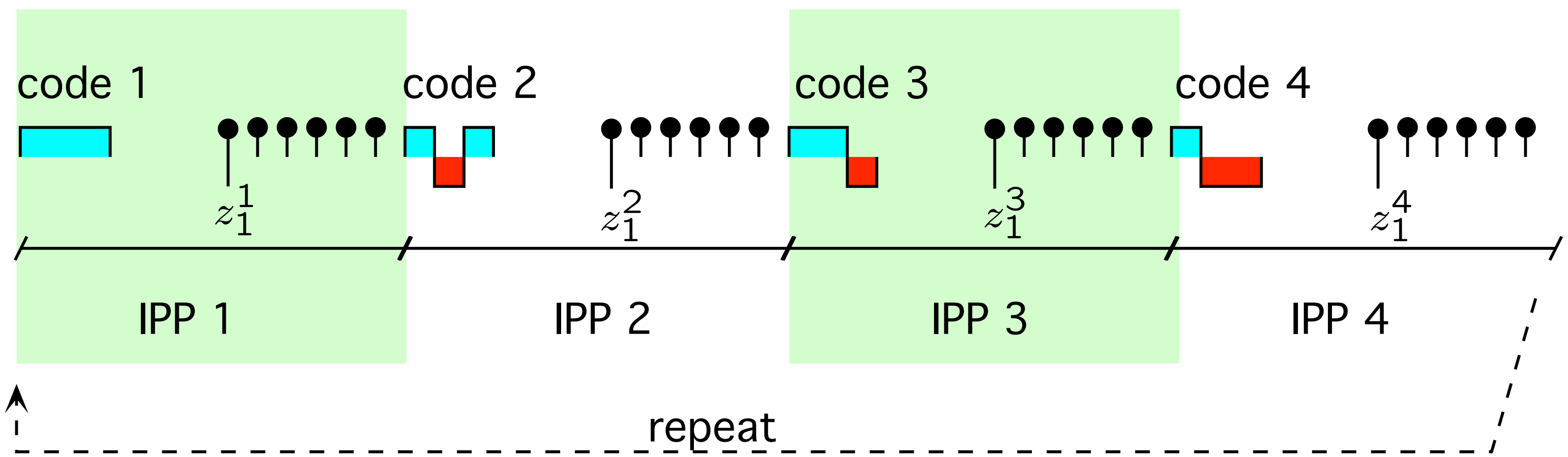
How they work:
the Coding and and
the Decoding
→

How it works:
THE CODING

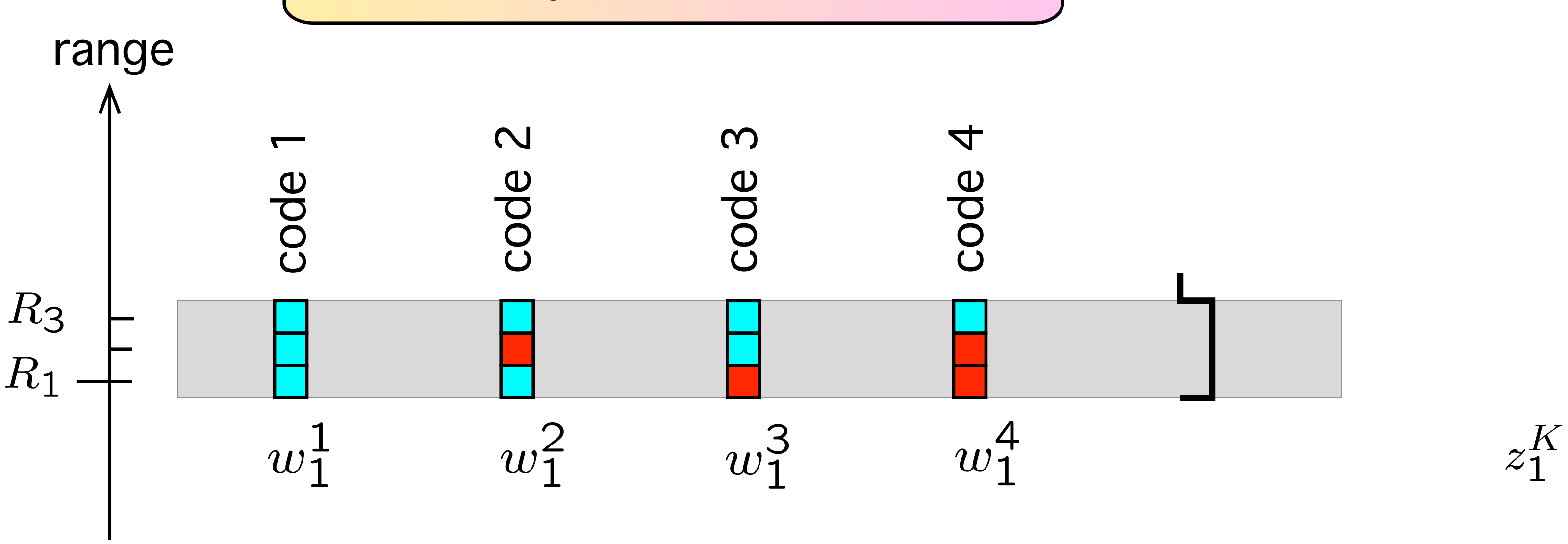
Transmit four 3-bit phase codes, each code received separately

Coding by flipping transmission phase

"3-bit alternating code"

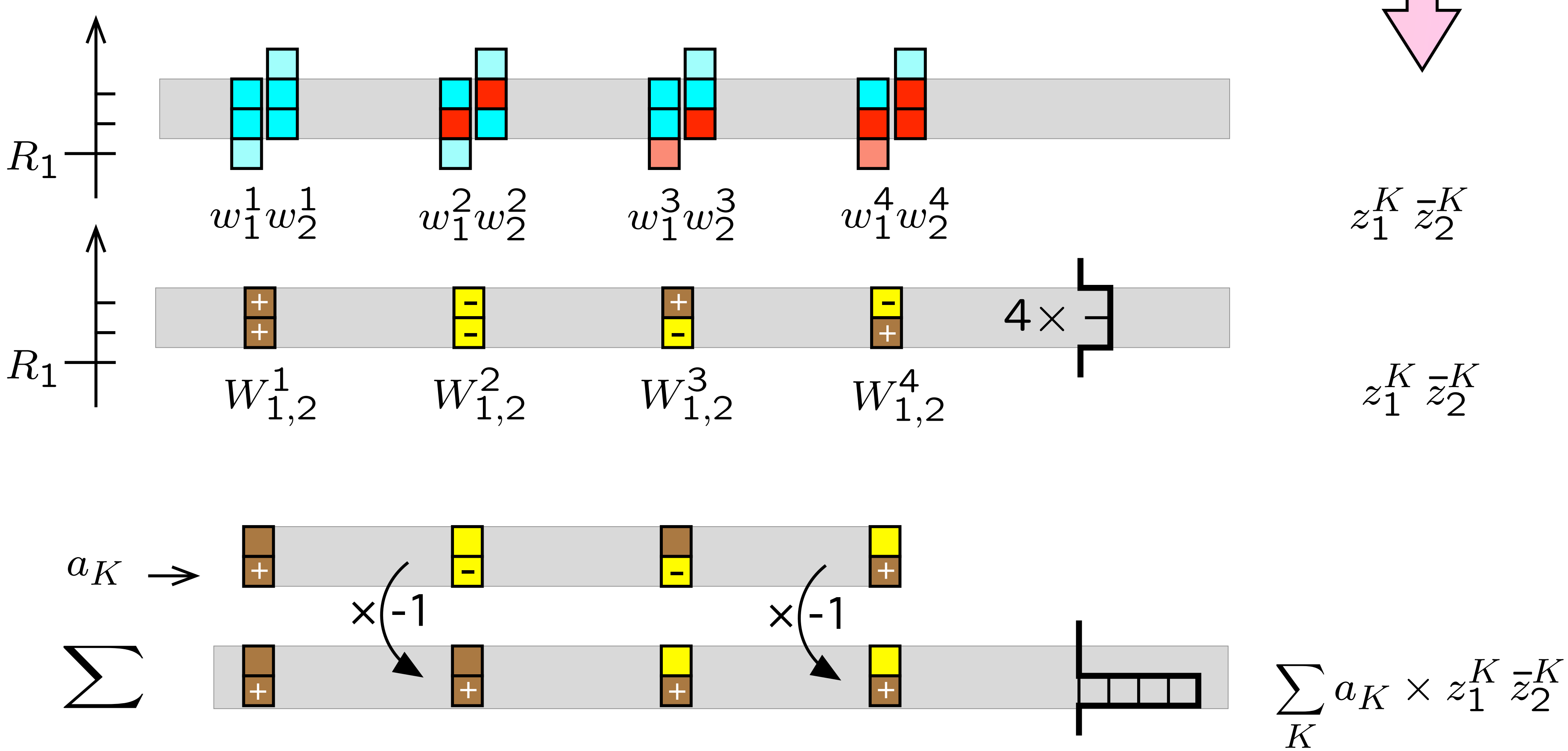


spatial weights of first samples



How it works:
THE DECODING

Decoding by taking linear combinations of the lagged products

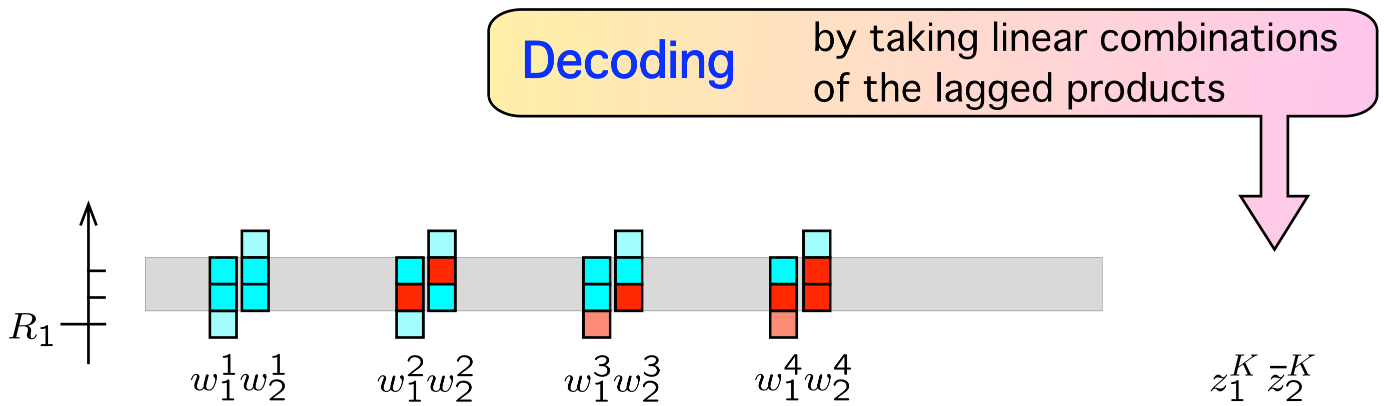


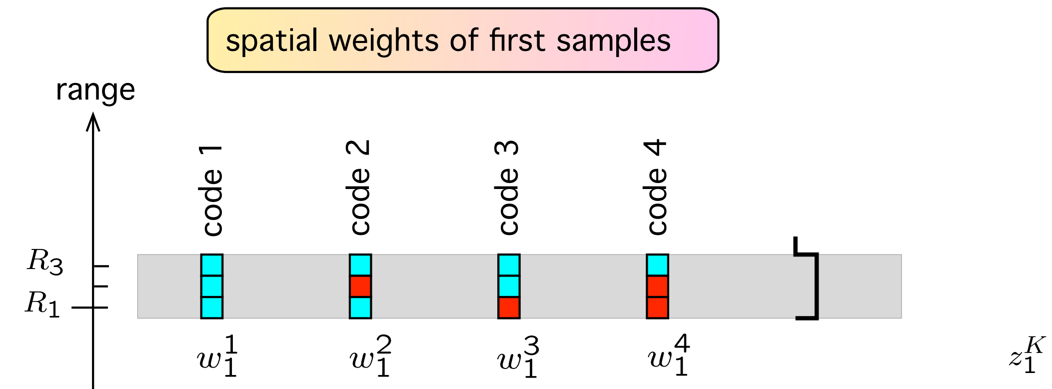
decoding lag 1, lowest range

❶ lag-1 product for each code K

❷ a suitable linear combination of the four lag-1 products

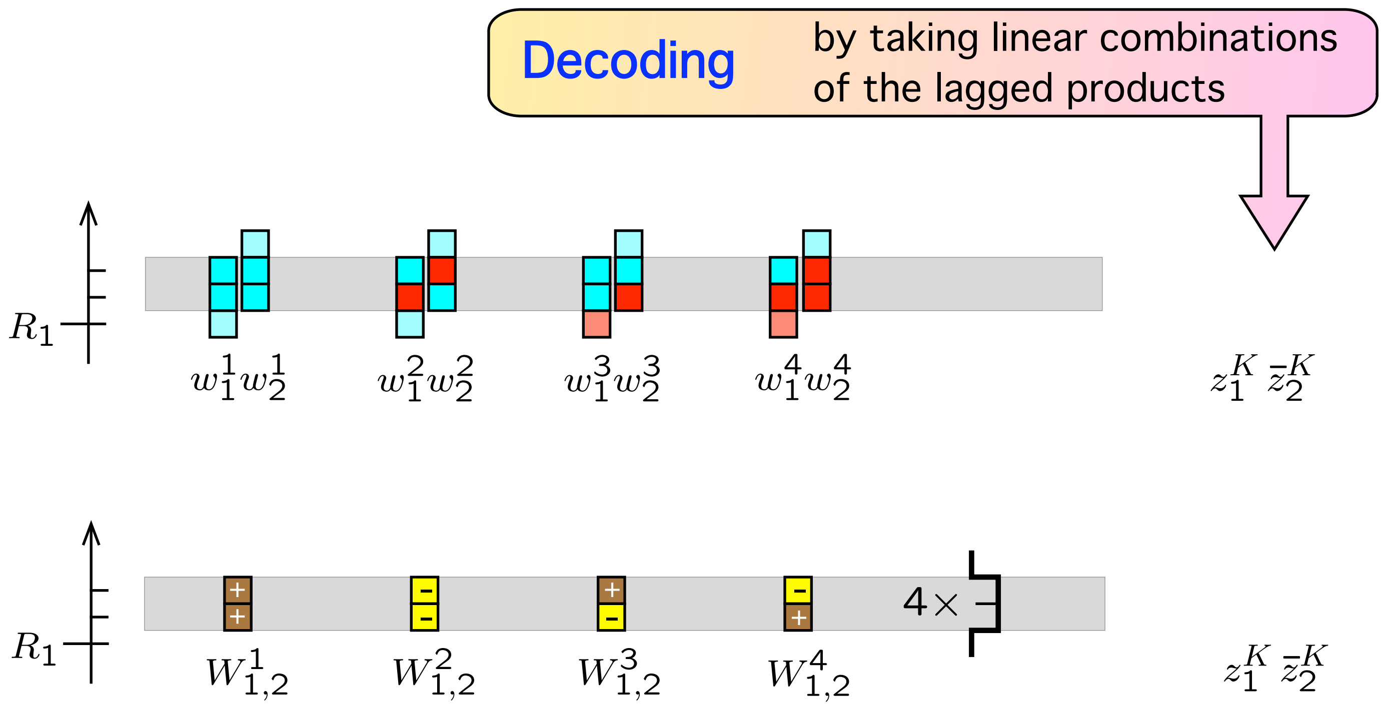
Decoding step 1





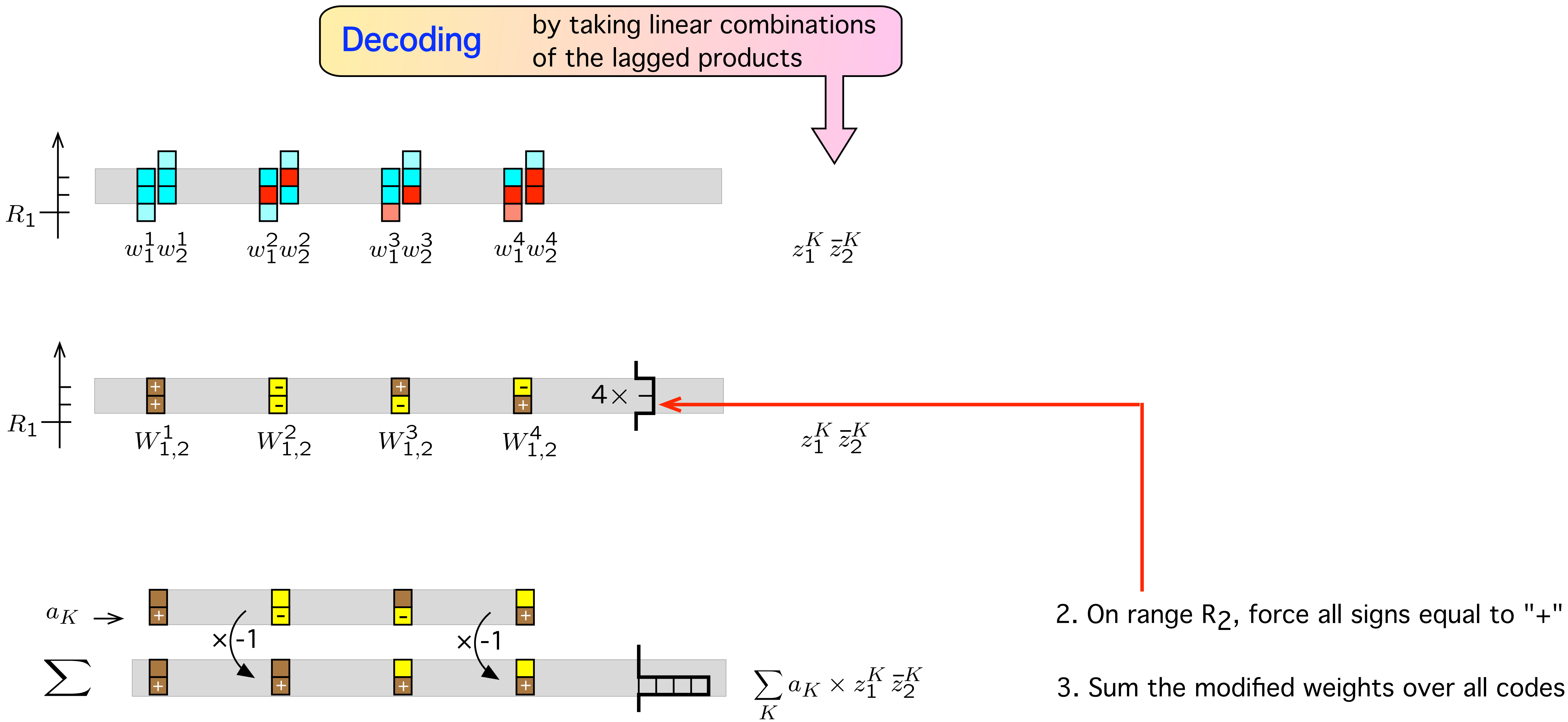
1. For each code K, form the weights of the lag=1 products

Decoding step I: form weights of the lagged products



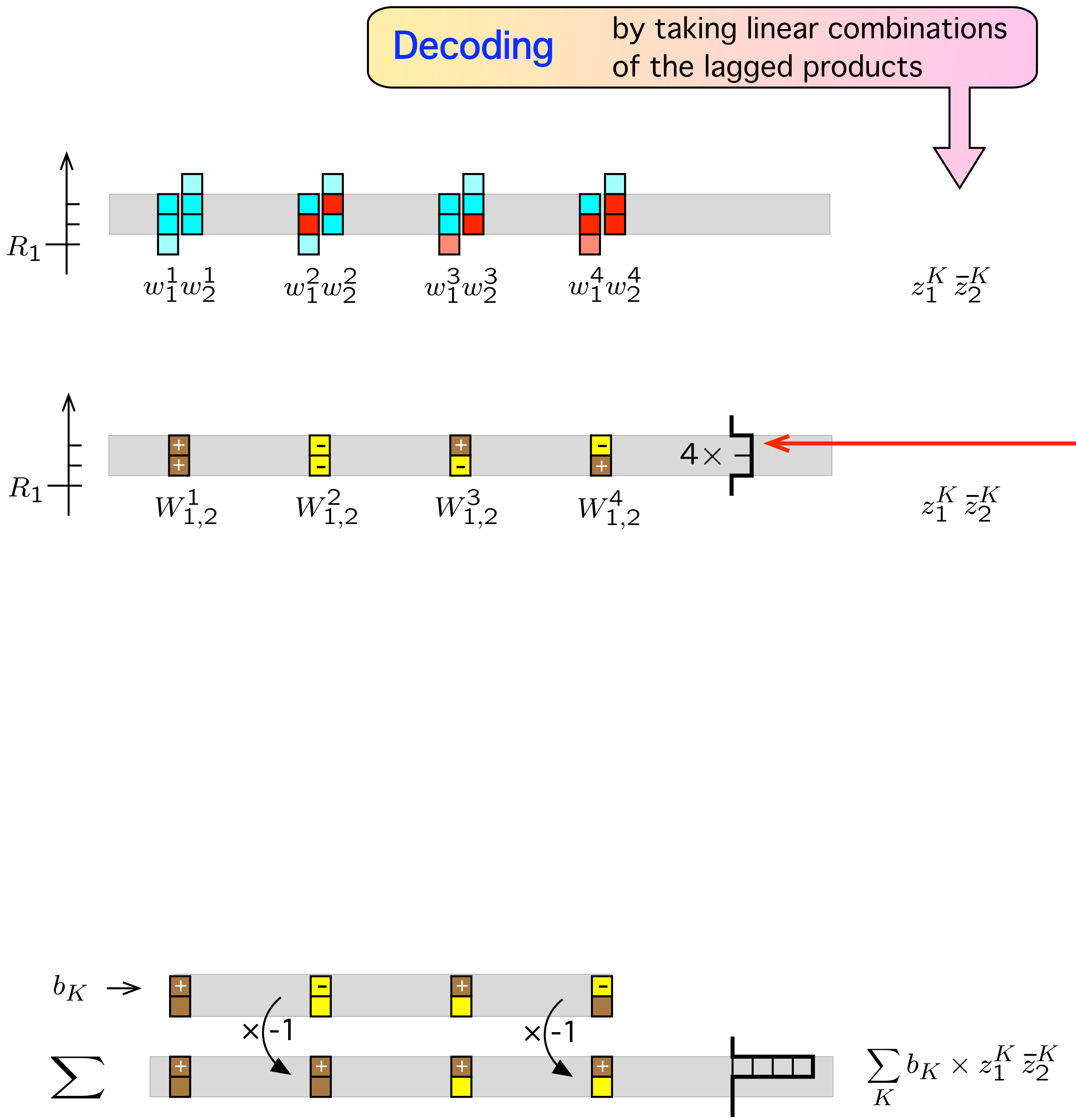
1. For each code K, form the weights of the lag=1 products

Decoding step IIA: form a suitable linear combination



Decoding step IIB: form another suitable linear combination

so as not to waste information



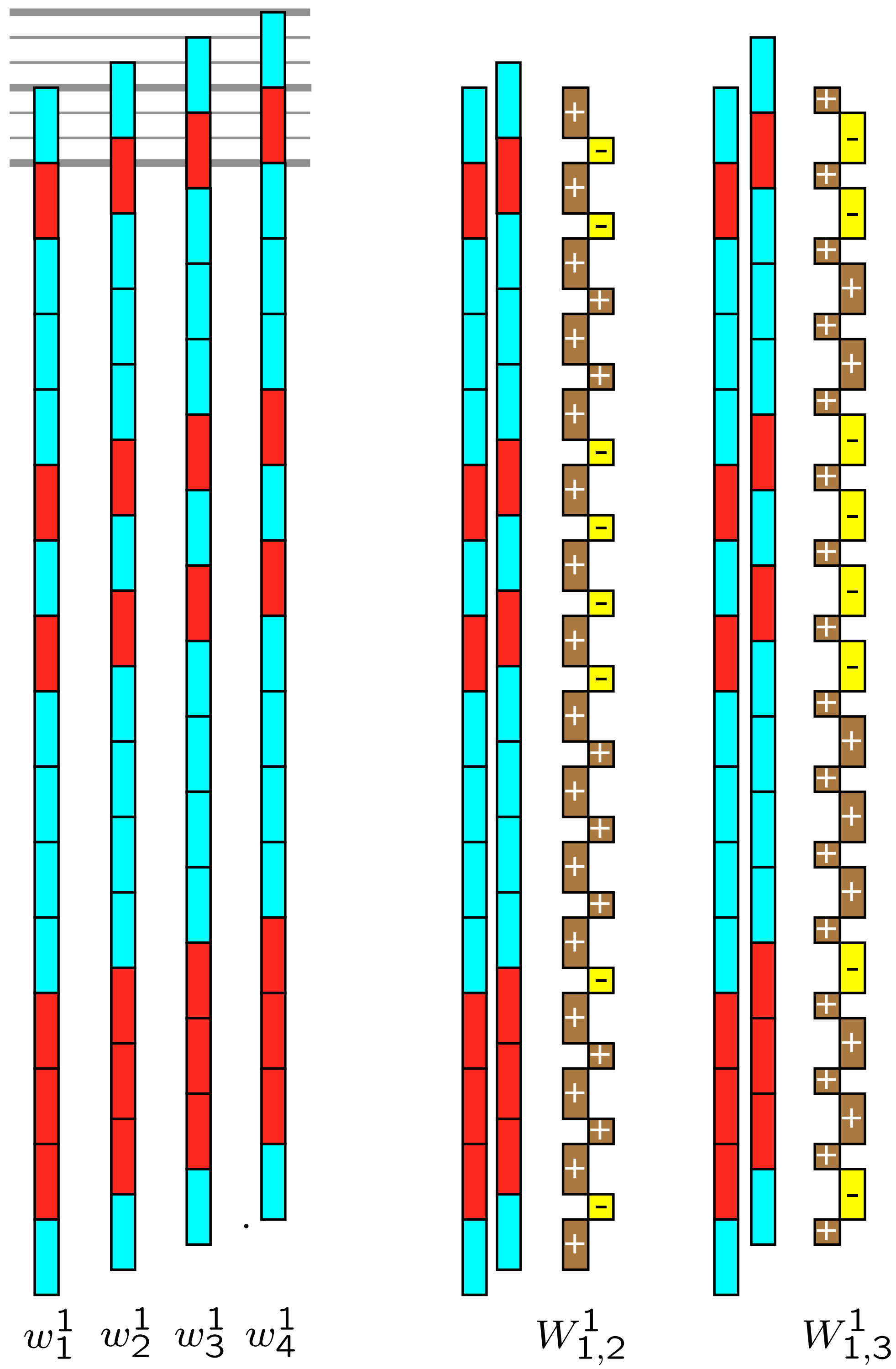
4. On range R_3, force all signs equal to "+"

5. Sum the modified weights over all codes

Our typical alternating code: 16-bit code, with fractional lags

Realistic case 16-bit code, "fractional lags"

```
 1: +-+++-+-----+++-
 2: +++++-+++----+++
 3: +--++--+++-++---
 4: ++-++----+-+---+
 5: +-+-+---++++-+++
 6: +++-+--+-++++++-
 7: +---+-++--+----+
 8: ++--+-+-+-+-+---
 9: +-++-----++-----
10: ++++---++++-+--+
11: +--+--+++-++-++-
12: ++-+--+---++++++
13: +-+---+-+--++--+
14: +++---++---+----
15: +------+-+--++++
16: ++------++---++-
17: +-++++----+-+-++
18: ++++++-++-+---+-
19: +--+++++++++++-+
20: ++-++++--+++-+--
21: +-+-+++-++-+--+-
22: +++-++++-+-++-++
23: +---++-+-----+--
24: ++--++--+---++-+
25: +-++-++--+---+-+
26: ++++-+++++--++--
27: +--+-+-++--+--++
28: ++-+-+-----++-+-
29: +-+--+--+-++++--
30: +++--+-+--++-+-+
31: +----+++-++-+-+-
32: ++---++-+++---++
```



Fractional lags:

lag increment =sampling interval = baud length / M

Improve range resolution without using more code bits, hence, without the need to use longer code cycle.

SOME PULSE SCHEMES